

## N O T I C E

THIS DOCUMENT HAS BEEN REPRODUCED FROM  
MICROFICHE. ALTHOUGH IT IS RECOGNIZED THAT  
CERTAIN PORTIONS ARE ILLEGIBLE, IT IS BEING RELEASED  
IN THE INTEREST OF MAKING AVAILABLE AS MUCH  
INFORMATION AS POSSIBLE

VARIATIONS IN TRANSPORT DERIVED FROM  
SATELLITE ALTIMETER DATA OVER THE GULF STREAM

SAI-81-278-WA

OPD TR 80-501-01

January 1981

Prepared by:  
Eugene Molinelli  
and  
Richard B. Lambert, Jr.



Prepared for:  
Mr. Robert E. Cheney  
National Aeronautics and  
Space Administration  
Goddard Space Flight Center  
Bldg 16W  
Greenbelt, Md.

(NASA-CR-166671) VARIATIONS IN TRANSPORT  
DERIVED FROM SATELLITE ALTIMETER DATA OVER  
THE GULF STREAM (Science Applications, Inc.,  
McLean, Va.) 54 p HC A04/MF A01 CSCL 08C

N81-25647

Unclas  
G3/48 28174

SCIENCE APPLICATIONS, INC.


1710 Goodridge Drive  
McLean, Virginia 22102  
(703) 821-4300

*SAI*

GODDARD SPACE FLIGHT CENTER  
**NASA CR 16667/**  
REQUEST FOR APPROVAL FOR PUBLICATION AND RELEASE

(See Reverse for Detailed Instructions, Reference to GFC 220.5, Approval and Special Requirements for GSFC Publications)

1. TITLE Variations in Transport Derived from Satellite Altimeter Data Over the Gulf Stream				
2. DOCUMENT NUMBER <del>XXXXXXXXXX</del>		Typed name and signature of Author(s) Eugene Molinelli and Richard Lambert		
3. APPROVAL REQUESTED FOR: <input type="checkbox"/> Presentation <input type="checkbox"/> Preprint (TM) <input type="checkbox"/> Journal Publication <input type="checkbox"/> Thesis (TM) <input type="checkbox"/> Formal NASA Report <input checked="" type="checkbox"/> CR Release <input type="checkbox"/> X-Document (Working Paper)		Science Applications, Inc. McLean, VA  NAS 5-25864 January 1981		
NOTE: COMPLETE FOR PRESENTATIONS, JOURNAL PUBLICATION AND PREPRINTS				
4. Name of organization or professional meeting, location and date held		5. Name of journal, proceedings, etc.		
6. PUBLICATION CONTENT CONSIDERATIONS: YES NO <input type="checkbox"/> <input checked="" type="checkbox"/> Contains Classified Material <input checked="" type="checkbox"/> <input type="checkbox"/> Uses International System of Units (SI) <input type="checkbox"/> <input checked="" type="checkbox"/> Describes Potentially Patentable Subject Matter  * Forward to (i.e., A new and useful process, product, mechanical and Patent Counsel, electrical arrangement of parts, or composition or matter,) Code 204		7. This paper has been prepared keeping specifically in mind trade secrets or suggestions of outside individuals or concerns which have been communicated to the Center in confidence, and does not violate any such disclosures, and has been examined to see that all participating groups have been given proper credit.		
6a. Typed name and signature of reviewer in Office of Patent Counsel		Primary Author		
APPROVALS AND APPROVAL AUTHORITY (See instructions on the reverse of this form)				
8a. Typed name and signature of Branch Head		<input type="checkbox"/> Approve <input type="checkbox"/> Disapprove	Comments	Code Date
8b. Typed name and signature of Division Chief <i>Eugene D. Salomonson</i>		<input checked="" type="checkbox"/> Approve <input type="checkbox"/> Disapprove	Comments	Code 920 Date 6/8/81
FOR EARLY DOMESTIC DISSEMINATION. The Division Chief <u>MUST</u> Also Indicate FEDD Applicability.		<input type="checkbox"/> YES <input checked="" type="checkbox"/> NO	This paper presents information having potential commercial application as outlined in GMI 2210.1.	
8c. Typed name and signature of Director of		<input type="checkbox"/> Approve <input type="checkbox"/> Disapprove	Comments	Code Date
8d. Typed name and signature Office of the Director		<input type="checkbox"/> Approve <input type="checkbox"/> Disapprove	Comments	Code Date
DOCUMENT RELEASE INSTRUCTIONS				
9. CONTRACTOR REPORTS - HIGH NUMBERED				
1. <input checked="" type="checkbox"/> Announce in STAR (no limitations on availability)		3. <input type="checkbox"/> Release by Originating Office Only		
2. <input type="checkbox"/> Announce in LSTAR (distribution limited as indicated) a. <input type="checkbox"/> U.S. Government Agencies and Contractors Only b. <input type="checkbox"/> U.S. Government Agencies Only c. <input type="checkbox"/> NASA and NASA contractors Only				



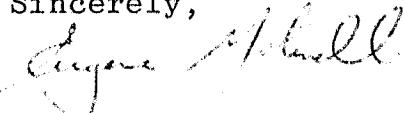
12 February 1981

Ms. Virginia Kendall  
Graphic Arts Branch, Code 253.1  
Goddard Space Flight Center  
Greenbelt, MD 20771

Dear Ms. Kendall:

Enclosed please find one unbound copy, suitable for reproduction, of our final report under NASA contract NAS5-25864. The report is entitled "Variations in transport derived from satellite altimeter data over the Gulf Stream" and has an SAI technical report number: SAI-81-278-WA.

Sincerely,



Eugene Molinelli

EM/rlw

## ABSTRACT

Variations in total change of sea surface height ( $\Delta h$ ) across the Gulf Stream are observed using SEASAT radar altimeter data.  $\Delta h$  is related to transport within the stream by a two layer model. Variations in  $\Delta h$  are compared with Knauss (1969) in which observed changes in transport were found to increase with distance downstream. No such increase is apparent since the satellite transports show no significant dependence on distance and typically vary widely from Knauss' values. Though most discrepancies are less than 50%, a few cases differ by about 100% and more. Several possible reasons for these discrepancies are advanced, including geoid error, but only two oceanographic contributions to the variability are examined, namely, limitations in the two layer model and meanders in the current. It is concluded that some of the discrepancies could be explained as changes in the density structure not accounted for by the two layer model. In some cases velocity changes associated with Gulf Stream meanders can contribute significantly to the discrepancies. It is suggested that a better density model be researched and that the effects of meandering be included.

## Section 1 INTRODUCTION

Cheney and Marsh (1980) showed how SEASAT altimeter data can be used to locate and characterize the Gulf Stream and Gulf Stream Rings. As part of that study, they mention a substantial variability in inferred Gulf Stream transport: "On time scales of a few days, surface transport indicated by the dynamic height difference across the Stream varied by nearly 30%, and over the entire 3-month period much larger fluctuations were observed, implying significant changes in total mass transport." In this paper we attempt to discern whether the variability of the Gulf Stream transport demonstrates a systematic spatial dependence.

## Section 2

### HEIGHT PROFILES FROM SEASAT ALTIMETER DATA

Figure 1 is reproduced from Cheney and Marsh (1980). It shows the relation between the measured radar range from the ocean surface to the satellite and the height of the sea surface above the geoid. All vertical distances in this analysis are expressed with respect to a reference ellipsoid. Values of range are provided every second, and represent averages over a small area (called a footprint). The footprint depends somewhat on sea state, but is typically 4 km wide by 7 km long. Footprint centers are about 7 km apart.

The position of the satellite's center of gravity is determined to within .08m during measurements from ground stations (Tapley et al., 1979). Between fixes, the satellite position is less certain, with a global average uncertainty of about 1.5m, (Cheney and Marsh, 1980). However, this uncertainty only affects horizontal scales greater than  $10^6$ m, so over scales of the order of the Gulf Stream, the error is well approximated by a tilt and a bias. More will be said on tilt determination later in this section.

The measured range is corrected for instrumental effects such as satellite attitude, time tag bias, gain, and obvious blunder points. Corrections are also made for environmental effects of the ionosphere and the dry and wet components of the troposphere. The signal that remains after these corrections is the height of the sea surface with respect to the reference ellipsoid ( $H_1$ ). The sea

surface height includes the cumulative effects of sea state, tides, geoid, slope currents, variations in the density of the water column and surface atmospheric pressure. Depending upon which of these components is of interest, the others have to be estimated or calculated and removed from the signal.

The component (h) which is due to ocean circulation, is given by

$$h = H_1 + D_{SWH} + D_T + D_B - GH \quad (2.1)$$

where  $D_{SWH}$  corrects for the effects of sea state,  $D_T$  corrects for tides,  $D_B$  accounts for atmospheric loading, and  $GH$  represents the geoid.

In correcting for sea state effects, two empirical relationships are used. The first determines the sea state from the spread of the leading edge of the radar pulse returning to the satellite. These values are expressed in terms of the significant wave height ( $H_{1/3}$ , i.e., the average height of the 1/3 highest waves) and are provided with the height data. The second relationship estimates the effect on the measured height of a given value of  $H_{1/3}$ . We use the approximation,

$$D_{SWH} = .05 H_{1/3} \quad (2.2)$$

(Jackson, 1979).

For the tidal component we use the solid earth and ocean tide corrections supplied with the data. The ocean tide is based on a model by Schwiderski (1978) and the solid earth tide is based on Melchior (1978).



Sea surface loading by atmospheric pressure (the barotropic correction) is also provided with the altimeter data. It is calculated from the simple relationship  $D_B = .009948 (P - 1013.3)$  given by Ronai (1979) where  $P$  is atmospheric pressure.  $P$  is determined from data obtained from Fleet Numerical Weather Central which is interpolated to values at satellite track positions.

The geoid is the last contribution to be removed. The geoid supplied with the altimeter data is a model of the component wavelengths greater than 1000 km. Large variations from this model occur over shorter wavelengths and can be as large as several meters in height over distances of 100 km - enough to overwhelm the oceanographic signal. In the present study, which is confined to the Gulf Stream west of the Sargasso Sea, a higher-resolution geoid (Marsh and Chang, 1972) is used. Geoid values in that model are provided on a 5' x 5' latitude-longitude grid. The 5' grid values are interpolated to sub-satellite points using an interpolation scheme developed at NASA/GSFC in Greenbelt Maryland (R. Cheney, personal communication).

This geoid, although the best available, is not without errors. The mean error, north of 25°N and away from the edges, as determined from a comparison with mean sea surface topography from GEOS-3 altimetry (Marsh et al, 1979), is about .5m, but individual errors can be greater near steep and complex topography and can occur over short distances. Some examples in the data examined here will be pointed out later in this section. The existence of such features requires that the user exercise extreme care in interpreting altimeter profiles.

Figure 2a shows a profile of  $\Delta h$  along a satellite track in the western North Atlantic. This profile still contains tilt and bias orbit errors, and geoid errors as well as a bit of noise apparent as  $\sim 0.1\text{m}$  rms scatter, but the Gulf Stream is already apparent. The noise can be reduced by applying a 3-point boxcar filter. The resulting smoothed profile is shown in Figure 2b.

The height change across the Gulf Stream depends upon the tilt of the combined orbit and geoid error. This can be estimated by fitting a straight line to a portion of the profile where no significant oceanographic signal is expected, as in the relatively quiescent central Sargasso Sea (Figure 2c). The new heights, with the tilt estimate removed, are shown in Figure 2d. All height profiles used in this study were produced in this way. The major uncertainties in these heights are due to the subjective manner of estimating tilt and the likelihood of geoid errors due to severe topography occurring in the region where the Gulf Stream flows. This is in contrast to the open Sargasso over which the tilt was determined and in which there is little topographic variation.

Seasat altimeter profiles from an eighteen day period (July 28 to August 15, 1978) were examined for spatial dependence in the variability of  $\Delta h$ . Eight profiles used in this study were generated by the above procedures at SAI (Figures 3a - 3h). Their positions are mapped in Figure 4. Other profiles generated at SAI were not used

because of excessive ambiguity in locating the Gulf Stream or insufficient intersection with both the Gulf Stream and the Marsh and Chang geoid. Two marginal profiles are included - orbits 449 and 493 (Figures 3a and 3d) - that show the difficulties sometimes encountered when interpreting the data. In the case of Figure 3a, the large increase across the Gulf Stream depends upon a straight line fit to a very short section just south of the stream. Both regions in that figure occur over seamounts in the New England chain and at least some of the peaks evident in the profile must be considered due to topographically influenced perturbations in the geoid. In the case of Figure 3d, the vertical excursions in the southern portions of the profile must be due, at least in part, to the rough topography in the Bahamas, the steps in the north due to the closeness of the Florida shore and only the large step about 27°N due to the Gulf Stream.

In other profiles geoidal perturbations are not necessarily less, but are more easily distinguishable from the Gulf Stream. An example is the isolated peak in the middle of Figure 3b (orbit 464). There is always the possibility that such residual geoidal effects may substantially distort measurements of  $\Delta h$ .

### Section 3

#### MODELS OF GEOSTROPHIC TRANSPORT

A profile of sea surface height ( $h$ ) with horizontal distance ( $l$ ) along the satellite track can be obtained from altimeter data as described above. In many applications the horizontal gradient of height ( $\Delta h$ ) is of interest. The gradient of sea surface height is related to the derivative of height along the track ( $\partial h / \partial l$ ) by the angle ( $\theta$ ) between the satellite track and the local contours of constant height.

$$|\underline{\nabla} h| = \frac{1}{\sin \theta} \frac{\partial h}{\partial l} \quad (3.1a)$$

The sea surface pressure gradient is related to the sea surface height gradient by the hydrostatic relation.

$$\underline{\nabla} P = g \rho \underline{\nabla} h \quad (3.1b)$$

Assuming geostrophic balance, a surface velocity can be calculated from the gradient of sea surface height

$$\underline{V}_g = \frac{g}{f \sin \theta} \frac{\partial h}{\partial l} \quad (3.2)$$

where  $V_g$  = magnitude of geostrophic velocity  
 $g$  = acceleration of gravity  
 and  $f$  = Coriolis parameter

The transport between two points along the satellite track is given by

$$T = \int_l \int_z V(z, l) \sin \theta \, dl \, dz \quad (3.3)$$

where  $z$  = depth  
 $T$  = transport  
 $V$  = velocity, a function of depth and distance along the track  
 $\phi$  = angle between the velocity and the satellite track, in general, a function of depth. For geostrophic flow, at  $z=0$ ,  $\phi=\theta$

In this relation velocity must be integrated from the bottom of the moving layer ( $z=-D$ ) to the sea surface ( $z=h$ ) as well as from the position  $\ell_1$  to position  $\ell_2$ . Thus some dependence of  $V$  on depth must be assumed. Two models for the depth dependence are considered here, a barotropic model and a simple baroclinic (two-layer) model.

In a geostrophic barotropic model  $V(z,\ell)$  equals  $V_g$  for every depth. Assuming further that the ocean depth is a constant ( $=D_0$ ) in the interval  $\ell_1$  to  $\ell_2$ , that the Coriolis parameter is also constant ( $=f_0$ ) in the interval and that  $h \ll D$  we get:

$$T_A = g \frac{D_0}{f_0} (h_2 - h_1) \quad (3.4)$$

where the subscript A refers to the assumptions above of constant depth, constant  $f$ , geostrophic, barotropic, and deep ocean, and  $h_1$ =height at position  $\ell_1$  and  $h_2$ =height at position  $\ell_2$ . Note that  $T_A$  is independent of the angle  $\theta$ .

A geostrophic baroclinic model can be approximated by a two layer model (e.g., Stommel, 1966) with density  $\rho_1$  overlying density  $\rho_2$ ,  $\rho_2 > \rho_1$ . In the two layer model the dense water is at rest which means that no pressure gradients may exist. In addition, in that model the thickness of the light water layer goes to zero, and the dense layer intersects the sea surface, at the western margin of the current. If we define D as the depth of the bottom of the lighter water and h as the height of the sea surface and set  $H=D+h$  then for a deep layer at rest

$$H = \left( \frac{\rho_2}{\rho_2 - \rho_1} \right) h.$$

Stommel's velocity in the light layer (modified for a transect not perpendicular to the average velocity) is given by

$$V(l) = \frac{g'}{f \sin \theta} \frac{\partial H}{\partial l} \quad (3.5)$$

where

$$g' = \left( \frac{\rho_2 - \rho_1}{\rho_2} \right) g$$

We assume that  $\phi$  is not a function of depth, so  $\phi = \theta$  at all depths. Then integrating equation 3.3 from  $Z=-D$  to  $h$  gives the transport:

$$T_B = \frac{g}{2f_0} \left( \frac{\rho_2}{\rho_2 - \rho_1} \right) (h_2^2 - h_1^2) \quad (3.6)$$

where the subscript B refers to the assumptions of geostrophic, two layer flow with the deep layer at rest,  $\rho_2 - \rho_1 \ll \rho_2$ ,  $f = \text{constant}$ , and  $\theta = \text{constant}$ . Note that  $T_B$  is also independent of the angle  $\theta$ .

Section 4  
COMPARISON OF SURFACE AND SATELLITE  
OBSERVED TRANSPORTS

4.1           SATELLITE TRANSPORTS

Using the two layer model of equation 3.6 with a value of the density term equal to 500 (after Stommel, 1966), transport values can be calculated for the eight altimeter profiles in Figure 3. These profiles, identified by orbit number and arranged in order of increasing distance downstream are listed in Table 1a. For the seven profiles, Table 1a gives both  $\Delta h$  and transport for profiles both before and after tilt corrections have been applied. Also included in Table 1a are values of  $\Delta h$  supplied by Robert Cheney of NASA (personal communication) for six of the same seven profiles as well as for eight other profiles. All these values are for ascending orbits (i.e., profiles obtained during traverses of the Gulf Stream from southeast to northwest). Six other profiles, provided from descending orbits, are listed separately in Table 1b. Profiles from descending and ascending orbits were kept separate throughout this report to test for any potential differences in Gulf Stream heights due to differences in ascending and descending orbits. No such discrepancy has been identified.

4.2           SURFACE TRANSPORTS

Knauss (1969) collected observations of Gulf Stream transport determined by two techniques. The first uses the observed density field over a horizontal



section to calculate the geostrophic shear. This section is combined with direct observations of current at one depth from neutrally buoyant floats to give absolute velocities perpendicular to the section and, hence, transport. The second uses profiling "transport" floats which measure the vertically averaged horizontal velocity. Knauss finds that, within 10%, the transports so determined are a well defined function of distance downstream from the Florida Straits. He notes an increase in transport of about 7% per 100 km. We can use his Figure 5 to express the relationship analytically

$$T_k = 29e^{\alpha y} \quad (4.1)$$

where  $T_k$  = transport as compiled by Knauss (in  $10^6 m^3/sec$ )  
 $\alpha$  =  $6.83 \times 10^{-4} km^{-1}$   
 $y$  = distance in km along the stream from a reference position of  $25.5^\circ N$ ,  $80^\circ W$

Knauss' first method depends upon the assumption of geostrophy, but the second method is less constrained. The difference between the results of the two methods is small. Recent work has shown that deep ocean currents are geostrophic to within the capacity to test the relationship (about 10%) when averaged over times of the order of 4 days or more (e.g., Bryden, 1977). Knauss' observed relationship amounts to a Gulf Stream transport change from  $30 \times 10^6 m^3 sec^{-1}$  to  $130 \times 10^6 m^3 sec^{-1}$ . That is a mean transport of  $80 \times 10^6 m^3 sec^{-1}$  with variations up to  $50 \times 10^6 m^3 sec^{-1}$ . This regional dependence is sufficiently large to account for all variations in Gulf Stream inferred by Cheney & Marsh (1980). Knauss' result will be used as a reference for transport values in comparisons made hereafter.

The Reference curve is plotted in Figures 5a and b along with transports calculated from SEASAT altimeter height profiles from Tables 1a and b, respectively. Some transports are shown for profiles both before and after attempts to correct for tilt errors. Though the tilt corrections, where recorded, usually bring the altimeter transports closer to the Reference curve, large discrepancies remain.

The first major discrepancy is that the satellite transports do not show a significant dependence on distance. The second is that the variability in transport from one satellite track to the next is much greater than the 10% quoted by Knauss. It is our view that the relationship determined by Knauss cannot be considered fortuitous and therefore that the discrepancies are due to uncertainties inherent in processing and interpreting the satellite data.

## Section 5

### CAUSES FOR DISCREPANCIES

Several contributions to the error in interpreting altimeter data can be advanced. As one factor, the residual errors in the 5' x 5' Marsh and Chang geoid may introduce sea surface slopes in the vicinity of the Gulf Stream very different from the slopes inferred in the Sargasso Sea. This can certainly be true in many cases especially where the Gulf Stream lies directly above the continental shelf break, or where bottom topography is especially rugged as it is along the New England Seamount Chain. See Section 2 for examples. However, this cannot be true in every case because the Gulf Stream often overlies smooth topography and because geoid error cannot be a factor in repeat mode satellite orbits where large (of the order of 30%) variations in Gulf Stream heights still are observed by Cheney and Marsh.

As a second factor, the determination of the geoid tilt in the Sargasso is subjective and may introduce some variability itself. The tilt correction applied to the profile depends upon which section of the height profile is used to fit the slope of a straight line. Mesoscale activity sometimes evident even in the "quiescent" Sargasso Sea, and breaks in the tilt of the geoid error make the final profile sensitive to the region chosen for the fit. Figure 5 shows the value of transport not only for the subjectively favored slope (solid dot) but for other reasonable choices (crosses). In some cases (i.e., orbits 478 and 622) the other choices may bring transports closer to the Reference values than the favored choice. In some cases, (i.e., orbits 464 and 550) the other choices yield values of

transport farther from the Reference line, and in some cases (i.e., orbits 579 and 636) the other choices provide little, if any, improvement. Though tilt subjectivity can explain discrepancies of the order of 30% it cannot explain the discrepancy in every case. Tilt correction subjectivity also accounts for the difference between NASA (open dot) and SAI (closed dot) heights and transports in Figure 5a, and in Table 1a.

Thirdly, the time scale of the satellite measurement of the Gulf Stream is so short (accomplished in about a minute) that a geostrophic balance may not be completely appropriate. Transient pressure fields with associated transient cross stream currents might be affecting the sea surface height measurements.

As a fourth factor, large scale non-geostrophic or quasi-geostrophic phenomena may be affecting the measurement. Gulf Stream meanders must have slightly unbalanced pressure gradient or Coriolis forces associated with them.

Finally, the two layer model used to infer transport from sea surface height changes may be less applicable at some times and locations than others.

If there is to be any hope of using satellite altimeter data to infer transport the geoid problem must be solved only by more extensive measurements and the time scale problem must introduce only errors of small size. We therefore concentrate our remaining efforts on understanding the effects of the last two problems listed above.

## 5.1 THE TWO LAYER MODEL

The two layer model approximates the baroclinic flow with a fast moving low density layer overlying a still, high density layer, which outcrops to the sea surface at one edge of the current. The density difference ratio ( $\rho_2/\rho_2 - \rho_1$ ) is assumed to be 500 after Stommel (1966). The model accuracy is limited in a number of ways: 1) the density difference ratio is only an approximation and may also have spatial and temporal variability; 2) the velocity shear is probably smaller, and more continuous than assumed; 3) the thickness of the light layer probably does not go to zero at the edge of the stream so that the surface velocity extends over a thicker layer making the transport greater. For interpreting Figure 5, we are concerned whether any discrepancies between the two layer and continuous models are a function of space (or time).

To test the two layer model we use a hydrographic section across the Gulf Stream in June from the Fuglister Atlas (Fuglister, 1960) to determine the actual density structure, and hence the actual geostrophic shear. The Gulf Stream is positioned between stations numbered 5296 and 5303 on that section. To make the comparison most favorable for the two layer model we selected the 10°C isotherm, which nearly outcrops at the northern edge of the stream, as our assumed level of no motion. With this assumption we can produce a sea surface height profile (Figure 6) as well as a numerically integrated transport value of  $35.7 \times 10^6 \text{ m}^3 \text{ sec}^{-1}$ . If we use the height profile of Figure 6 and the two layer model we get a two layer transport value of  $11.3 \times 10^6 \text{ m}^3 \text{ sec}^{-1}$ , which is 68% less than the value which the continuous density information gives.

The two layer model used here assumes a density difference ratio of 500 which is not necessarily consistent with a 10°C reference depth. A more appropriate value of the ratio was determined to be 200  $\pm$  145. This was determined from 243 hydrographic stations from the NODC data set (through 1974) for the months July through September in the region of the Gulf Stream from 30 to 40°N and west of 70°W. The average density of the deep layer was calculated for the depth interval from the 10°C isotherm depth (1000m or less) to 2000m. With this value of the ratio, the two layer model now gives a transport of  $4.5 \times 10^6 \text{ m}^3 \text{ sec}^{-1}$  which is 87% less than the numerically integrated value. Therefore the two layer model, even under the most ideal comparison, must be considered a poor approximation, in terms of absolute transport, to the actual stratification.

Another stringent test of the two layer model would be to compare its assumption of a level of no motion near the 10°C isotherm, to the level of no motion required to give the Reference result of  $81.5 \times 10^6 \text{ m}^3 \text{ sec}^{-1}$  for the position of the Fuglister section. The Reference result requires a level of no motion deeper than 2000m which corresponds to an isotherm less than 4°C. This isotherm comes no closer to the surface than 1000m at the Gulf Stream's northern edge, inconsistent with the two layer model assumption that the dense water outcrops to the sea surface at the northern edge. Therefore for this reason also, more realistic descriptions of geostrophic shear will be necessary if one is to use sea surface height profiles to calculate absolute transports.

A percent error in absolute transport that is constant with position and time will not explain the

deviations depicted in Figure 5. Therefore we need to know if the inadequacy of the two layer model changes with position and time. We can make use of another Fuglister section, this one along 66°W, which crosses the Gulf Stream in September between stations 5189 and 5197. According to Knauss the transport here is  $147 \times 10^6 \text{ m}^3 \text{ sec}^{-1}$ . Once again, this requires a level of no motion near 2000m and hence colder than 4°C. Table 2 shows the results. The transport calculated relative to 2000m compares favorably with the observed transport. However, the two layer model vastly under-estimates the transport and in this example shows a decrease in transport with distance, in opposition to the observations.

Thus the error of the two layer model is extremely sensitive to changes in the density field. The inadequacies of the two layer model alone can give rise to the large discrepancies between satellite-derived transports and observed transports illustrated in Figure 5. This does not rule out any other source for the errors, but merely indicates that changes in the density field produces variability of the correct order.

## 5.2 MEANDERS

In a meander with radius of curvature R and velocity V the equation of motion is

$$\frac{1}{\rho} \nabla P = fV + \frac{V^2}{R} \quad (5.1)$$

where  $R$  is negative for anticyclonic motion and positive for cyclonic motion, and  $\nabla P$  is the pressure gradient (e.g., Von Arx, 1962). The equation represents a deviation from the geostrophy equation which consists of the first two terms alone. We can combine equations 3.1a, 3.1b and 3.2 to write the equality.

$$\frac{1}{\rho} \nabla P = \frac{g}{\sin \theta} \frac{\partial h}{\partial \ell} = fV_g \quad (5.2)$$

where  $V_g$  is the geostrophic velocity used in the previous calculations of transport. Then Equation 5.1 becomes

$$fV_g = fV + \frac{V^2}{R} \quad (5.3)$$

For a given height profile and radius of curvature,  $V$  can be found from the quadratic formula constrained by the necessity for the velocity to go to zero as the height gradient goes to zero.

$$V = \frac{Rf}{2} + \frac{1}{2} \left( \frac{R}{|R|} \right) \sqrt{R^2 f^2 + 4RfV_g} \quad (5.4)$$

$R$  can be estimated from analyses of the Gulf Stream such as the Monthly Summary issued by the Naval Oceanographic Office. For a typical anticyclonic meander  $R$  is on the order of  $-3 \times 10^5 \text{m}$ , although in some areas stronger meanders have radii down to about  $-10^5 \text{m}$ .



We can examine the effect of a 300 km radius meander on the transport discrepancies by using orbit 464. There the average surface geostrophic velocity,  $V_g$ , is  $1.20 \text{ m sec}^{-1}$  assuming the satellite track crosses the Gulf Stream at an angle of  $75^\circ$ , and  $f$  is  $8.34 \times 10^{-5} \text{ sec}^{-1}$ . An anticyclonic meander with a 300 km radius of curvature would mean that the actual velocity there were given by equation 5.4, thus  $V = 1.26 \text{ msec}^{-1}$ . This  $.06 \text{ msec}^{-1}$  difference is only a 5% increase in velocity over the geostrophic. For the simple two layer model of a meander which does not change direction with depth, the  $.06 \text{ msec}^{-1}$  must be constant within the upper layer across the entire width of the stream. This constitutes a transport increase which is also 5%. Thus, a meander with a 300 km radius of curvature differs in sea surface velocity from a straight current by 5% and 5% in transport.

The effects of meanders on sea surface velocity at other values of the radius of curvature and geostrophic velocity are shown in Figures 7a and 7b. For a fixed value of  $V_g = 1.5 \text{ msec}^{-1}$  (typical Gulf Stream geostrophic velocity), the actual velocity will depend upon the radius of curvature. Figure 7a shows that for the case of anticyclonic meanders (negative radii) with curvature radii of 200 to 100 km, the change on sea surface velocity can be  $.16$  to  $.46 \text{ msec}^{-1}$  (11 to 30%) with the same order effects on transports. For a fixed value of  $R = -150 \text{ km}$ , Figure 7b shows that an increase of the geostrophic velocity from  $1.5$  to  $2.5 \text{ msec}^{-1}$  will cause the actual velocity to differ by  $.23$  to  $.90 \text{ msec}^{-1}$ , i.e., 15 to 36%. Once again the effect on transport may be on the same order.

To elucidate the effect of meandering on sea surface height, in both the along stream and cross stream directions, a more sophisticated model of meanders, must be employed which allows for continuity, and conservation of energy and momentum, as well as a balance of forces (e.g., Chew, 1974). Meanders also have significant effects on density gradients in the upper 1200m (Newton, 1978) so a proper meander model may be able to provide information to an improved density model as required by the previous section. Further work on this problem is in order.

## Section 6

### CONCLUSION

The sea surface height change across the Gulf Stream, as determined from satellite radar altimeter data, varies with location in the stream and also with time. Using a two layer model to transform satellite-determined sea surface height changes to transport values, it is found that transports within the Gulf Stream deviate from Reference values. The deviations from the direct observations are on the order of 50%.

A main reason for the discrepancy is the inability of the two layer model to reflect the variable density structure within the stream. Other effects that can contribute significantly to the discrepancies are geoid error, the subjectivity of the tilt error determination, and meanders in the current. Better models of the density structure combined with sophisticated models of meandering could greatly increase the utility of satellite altimeter measurements once the geoid is improved. Until the oceanographic models are improved, sea surface height changes can not be used to infer either the absolute or the relative variations in transport.

## ACKNOWLEDGEMENTS

This work was performed under NASA contract NAS 5-25864, as SAI project #1-424-00-800. Fruitful discussions were held with Bob Cheney, especially, and Jerry Hebenstreit and Steve Patterson. Programming assistance by Carol Hulick, figure drafting by Millie Coulmas and typing by Sharon Rubenstein and Jeanne Haynes are also gratefully acknowledged.

## REFERENCES

- Cheney, R. E. and J. G. Marsh, Seasat altimeter observations of dynamic ocean currents in the Gulf Stream Region, J. Phys. Oceanogr., in press, 1980.
- Chew, F., The turning process in meandering currents: a case study, J. Phys. Oceanogr., 4, 27-57, 1974.
- Fuglister, F. C., Atlantic Ocean Atlas of Temperature and Salinity Profiles and Data from the International Geophysical Year of 1957-1958, Woods Hole Oceanographic Institution, Woods Hole Mass., 1960.
- Jackson, F. C., The reflection of impulses from a nonlinear random sea, J. Geophys. Res., 84, 4939-4943, 1979.
- Knauss, J. A., A note on the transport of the Gulf Stream, Deep Sea Res., 16, 117-123, 1969.
- Marsh, J. G. and E. S. Chang, 5' detailed gravimetric geoid in the northwestern Atlantic Ocean, Mar. Geodesy, 1, 253-261, 1978.
- Marsh, J. G., T. V. Martin, J. J. McCarthy and P. S. Chovitz, Estimation of mean sea surfaces in the North Atlantic, The Pacific and the Indian Ocean using GEOS-3 altimeter data, NASA/GSFC TM 79704, 1979.
- Melchior, P., The Tides of the Planet Earth, Pergamon Press, New York, 1978.
- Newton, C. W., Fronts and wave disturbances in Gulf Stream and atmospheric jet stream, J. Geophys. Res., 83, 4697-4706, 1978.
- Ronai, P., Seasat interim geophysical data record (IGDR) users handbook, initial version-altimeter, Jet Propulsion Laboratory Report No. 622-97, Pasadena, Ca., 1979.
- Schwiderski, E. W., Global Ocean Tides, Part I: A Detailed Hydrodynamical Interpolation Model, NSWC/DL TR-3866, Naval Surface Weapons Center, Dahlgren Va., 1978.
- Stommel, H., The Gulf Stream, 2nd ed., University of California Press, Berkeley, 1966.

Tapley, B. D., G. Born, H. Hagar, J. Lorell, M. Parke,  
J. Diamante, B. Douglas, C. Goad, R. Kolenkiewicz,  
J. Marsh, C. Martin, S. Smith, W. Townsend, J. Whitehead,  
M. Byrne, L. Fedor, D. Hammond and N. M. Mognard,  
Seasat altimeter calibration: initial results, Science,  
204, 1410-1412, 1979.

von Arx, W. S., An Introduction to Physical Oceanography,  
Addison-Wesley, Reading Mass., 1962.

## TABLE CAPTIONS

- Table 1a    Summary of ascending orbits used to calculate Gulf Stream heights and then transports via the two layer model.  $\Delta h$  is height change across the Gulf Stream. The raw value refers to profiles before tilt correction, the favored value refers to profile corrected for tilt at SAI and NASA refers to values provided by Bob Cheney.
- Table 1b    Same as 1a but for descending orbits.
- Table 2    Comparison of the two layer model with other determinations of transport. The two layer model gives absolute values that are too low and relative values that, in this case, show an opposite trend.

TABLE 1a ASCENDING ORBITS

ORBIT #	GULF STREAM POSITION		$f$ ( $10^{-5}\text{sec}^{-1}$ )	DISTANCE (km)	$\Delta h$ (cm)			TRANSPORT (Sv)		
	LAT( $^{\circ}$ N)	Lon( $^{\circ}$ W)			NASA	RAW	FAVORED	NASA	RAW	FAVORED
493	26.5	79.5	6.49	228	-	226	174	-	193	114
579	31.3	79.1	7.49	810	140	173	119	64	98	46
622	32.0	78.1	7.70	933	160	174	159	82	96	80
665	32.9	77.1	7.90	1070	250	-	-	194	-	-
464	34.0	76.0	8.13	1237	95	90	119	27	24	43
507	35.0	75.0	8.34	1381	75	-	-	17	-	-
550	36.0	74.2	8.55	1514	130	125	145	48	45	60
636	37.0	72.3	8.75	1726	195	167	191	107	78	102
679	37.4	70.8	8.83	1883	120	-	-	40	-	-
478	37.6	69.4	8.95	2029	195	150	175	104	62	84
521	38.0	68.0	8.95	2159	205	-	-	115	-	-
564	38.0	67.0	8.95	2247	100	-	-	27	-	-
650	38.0	63.8	8.95	2536	220	-	-	133	-	-
693	38.4	62.7	9.03	2641	190	-	-	98	-	-
449	38.7	62.5	9.09	2680	200	98	295	108	26	235



TABLE 1b ASCENDING ORBITS

ORBIT #	GULF STREAM POSITION		$f$ ( $10^{-5}\text{sec}^{-1}$ )	DISTANCE (km)	$\Delta h$ (cm)			TRANSPORT (Sv)		
	LAT( $^{\circ}$ N)	LON( $^{\circ}$ W)			NASA	RAW	FAVORED	NASA	RAW	FAVORED
601	36.0	74.0	8.55	1532	100	-	-	29	-	-
644	37.5	71.0	8.85	1862	140	-	-	54	-	-
687	37.5	69.5	8.85	1998	130	-	-	47	-	-
443	37.5	69.0	8.85	2066	130	-	-	47	-	-
529	38.5	66.0	9.05	2350	175	-	-	83	-	-
572	37.5	65.0	8.85	2447	115	-	-	37	-	-

TABLE 2					
<u>SECTION</u>	<u>OBSERVED</u> <u>TRANSPORT*</u>	<u>TRANSPORT*</u> <u>wrt 2000m</u>	<u>h<sub>0</sub>+wrt</u> <u>2000m</u>	<u>TRANSPORT*</u> <u>2-LAYER MODEL</u>	<u>RELATIVE</u> <u>ERROR</u>
36°N	82	69	1.18	40	42%
66°W	147	148	.82	19	87%

\* transport in Sverdrups, 1 Sv = 10<sup>6</sup>m<sup>3</sup>sec<sup>-1</sup>

+ h<sub>0</sub> = h<sub>2</sub> - h<sub>1</sub>

## FIGURE CAPTIONS

- Figure 1      An illustration of the components included in the equation to determine sea surface height from a measured radar range. See text for more details. This figure is modified from Cheney and Marsh, 1980.
- Figure 2a     A profile of sea surface height above the geoid after all deterministic corrections. The geoid used is the Marsh & Chang (1978) 5' geoid. The Gulf Stream, measurement noise and a tilt error are all apparent.
- Figure 2b     Same as Figure 2a after smoothing with a 3 point box-car filter to reduce noise.
- Figure 2c     Same as Figure 2b with a straight line drawn which is fitted between latitudes  $27.7^{\circ}\text{N}$  and  $32.7^{\circ}\text{N}$  in order to estimate the tilt error.
- Figure 2d     Sea surface heights after correction for tilt error. Asterisks appear above points selected as Gulf Stream boundaries. H gives the height in meters and X gives the distance in kilometers from the left edge of the plot.

Figure 3a-h Profiles of sea surface height for eight satellite tracks after all corrections, smoothing and tilt removal. The Gulf Stream appears in each. Some features, e.g. the isolated peak in Figure 3b, are due to residual geoid error. Profiles are identified by orbit number, time and date above figure and latitude and longitude along distance axis.

Figure 4 Map of the eight satellite tracks whose profiles appear in Figure 3. Tracks are identified by orbit number.

Figure 5a Plot of transport as a function of distance along the Gulf Stream. The thick diagonal line marks the reference relationship based on Knauss (1969). The parallel thin lines form a 10% envelope within which all of Knauss' observations fall.

Transports based on satellite measurements are denoted by the various symbols and the vertical lines. Satellite height is converted to transport using the two layer model, and labelled by orbit number. The closed dots are based on the eight corrected profiles in Figure 3 while the short horizontal bar denotes transport before removal of the tilt estimate. A vertical line connects the two values. The crosses denote transports derived from profiles after different estimates of the tilt have been removed. The open dots are based on height changes across the Gulf Stream provided by Bob Cheney of NASA.

The satellite based values of transport do not show the same trend and vary widely from the reference values. Because of features due to geoid errors, transports based on orbits 493, 665 and 449 are somewhat questionable, but even without these there is not a significant trend with distance.

Only ascending orbits are contained here.

Figure 5b      Same as Figure 5a but for descending orbits.

Figure 6      Sea surface height along a hydrographic section (Fuglister, 1960) across the Gulf Stream assuming a level of no motion approximately at the depth of the 10°C isotherm. This height profile can be used to check the transport from the two layer model against the transport using the actual density field. See text and Table 2 for comparison results.

Figure 7a      Deviation from geostrophic velocity for anti-cyclonic meanders with varying radii of curvature. The assumed geostrophic speed is held constant at  $1.5 \text{ m sec}^{-1}$ . For radii in the 100 to 200 km range the deviations are significant (11 to 30%).

Figure 7b      Deviation from geostrophic velocity for anti-cyclonic meanders with varying values of the geostrophic velocity (hence varying values of the sea surface height gradient). The assumed radius of curvature is held constant at 150 km. For sea surface height gradients typical of the Gulf Stream (geostrophic velocities from 1.5 to 2.5m sec<sup>-1</sup>) the deviations are significant (15 to 36%).

# SATELLITE ALTIMETER MEASUREMENT

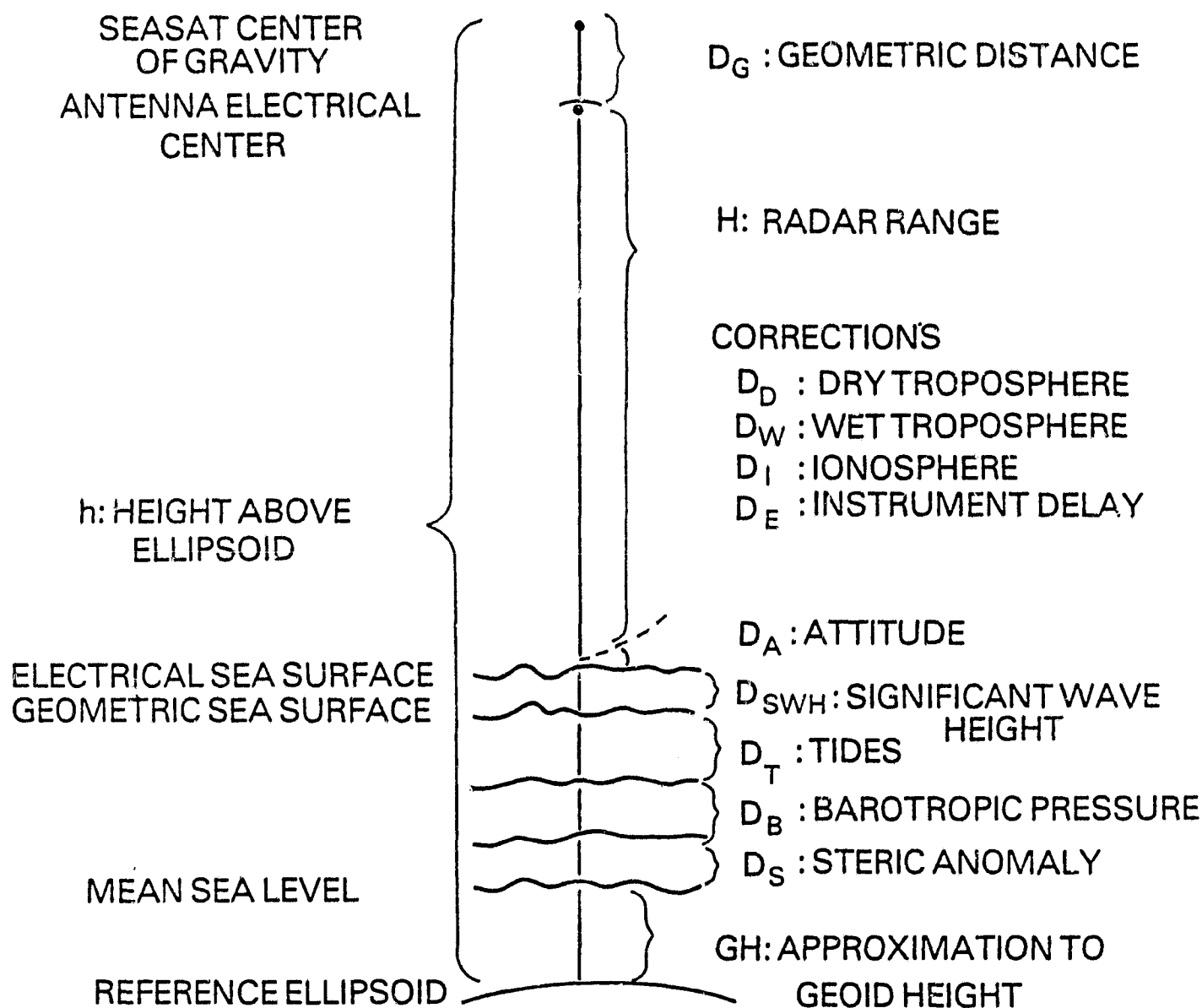


FIGURE 1

ORBIT 550  
11.09 - 11.19 3/8/78

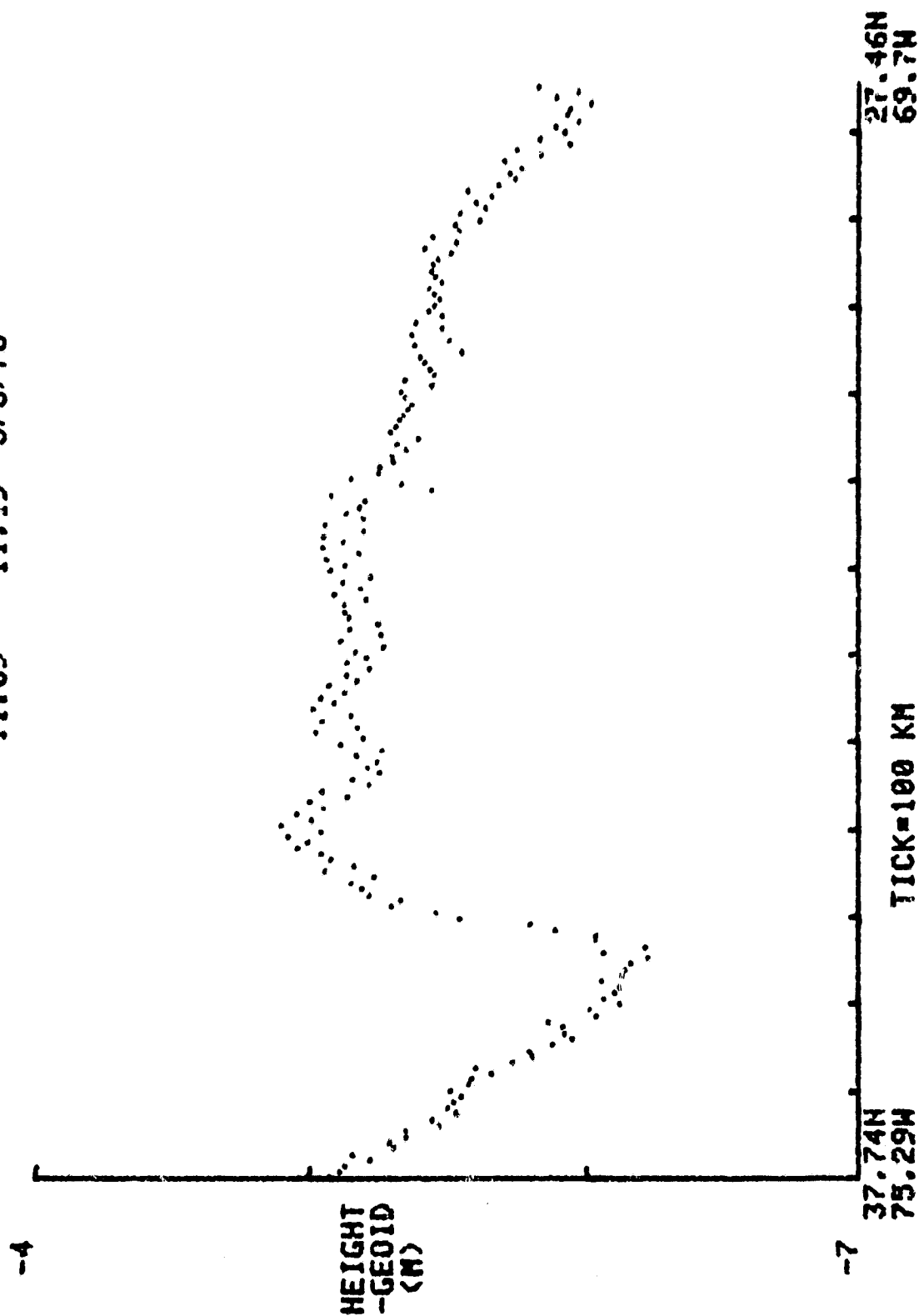


FIGURE 2a

ORIGINAL PAGE 1  
OF POOR QUALITY



ORBIT 550  
 11.09 - 11.19 3/8/78

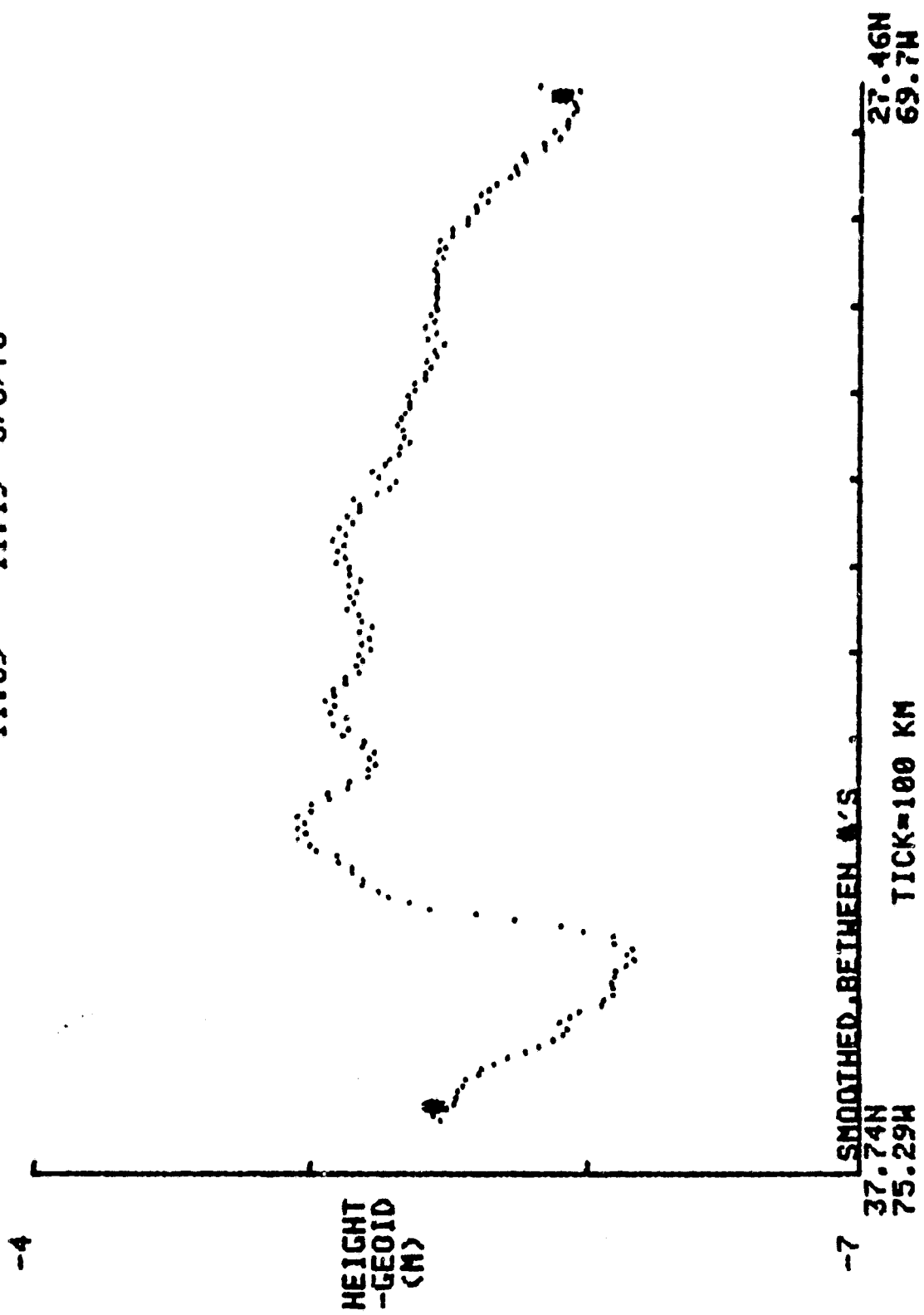


FIGURE 2b

ORBIT 550  
11.09 - 11.19 3/8/78

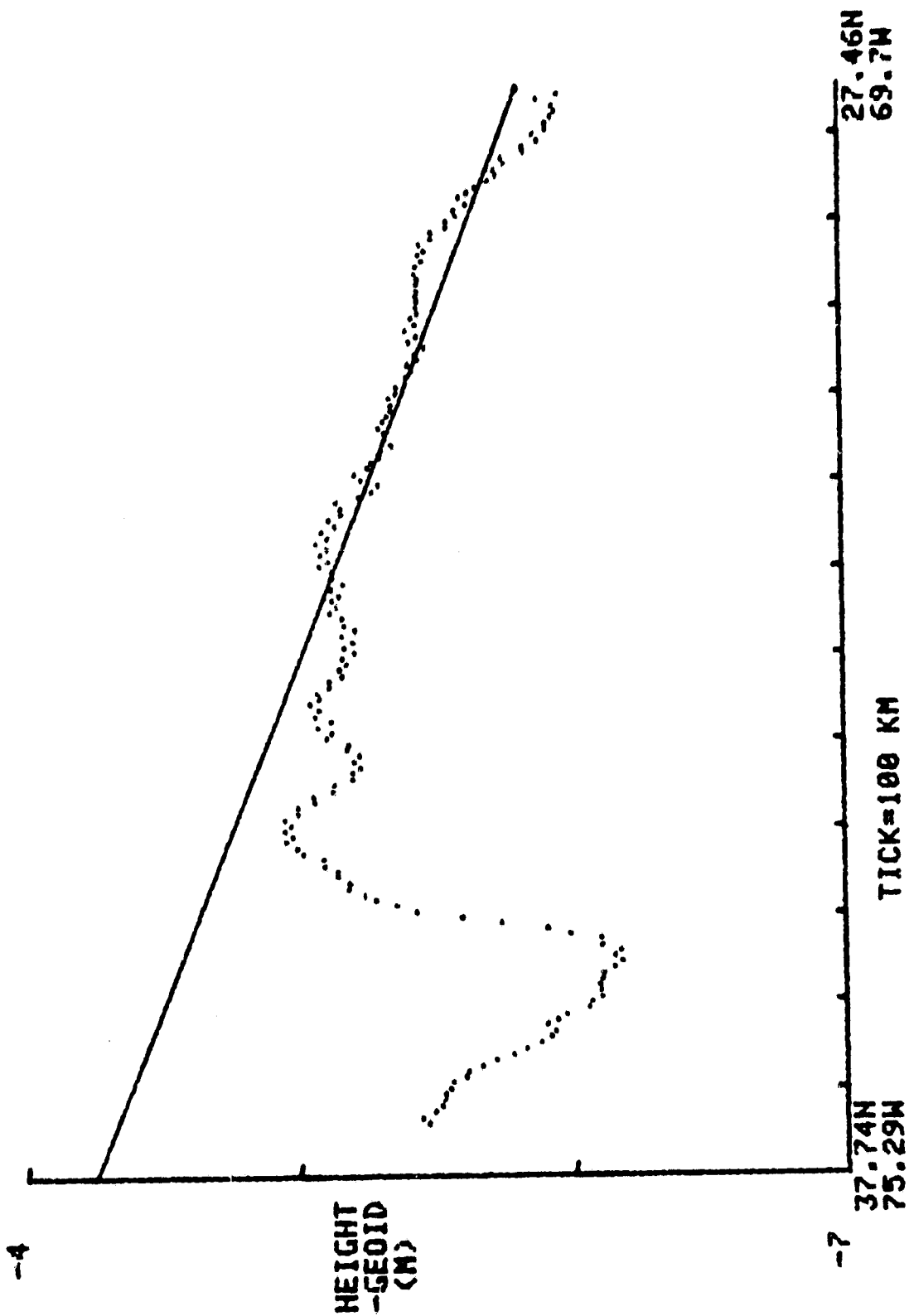


FIGURE 2c

ORBIT 550  
11.09 - 11.19 3/8/78

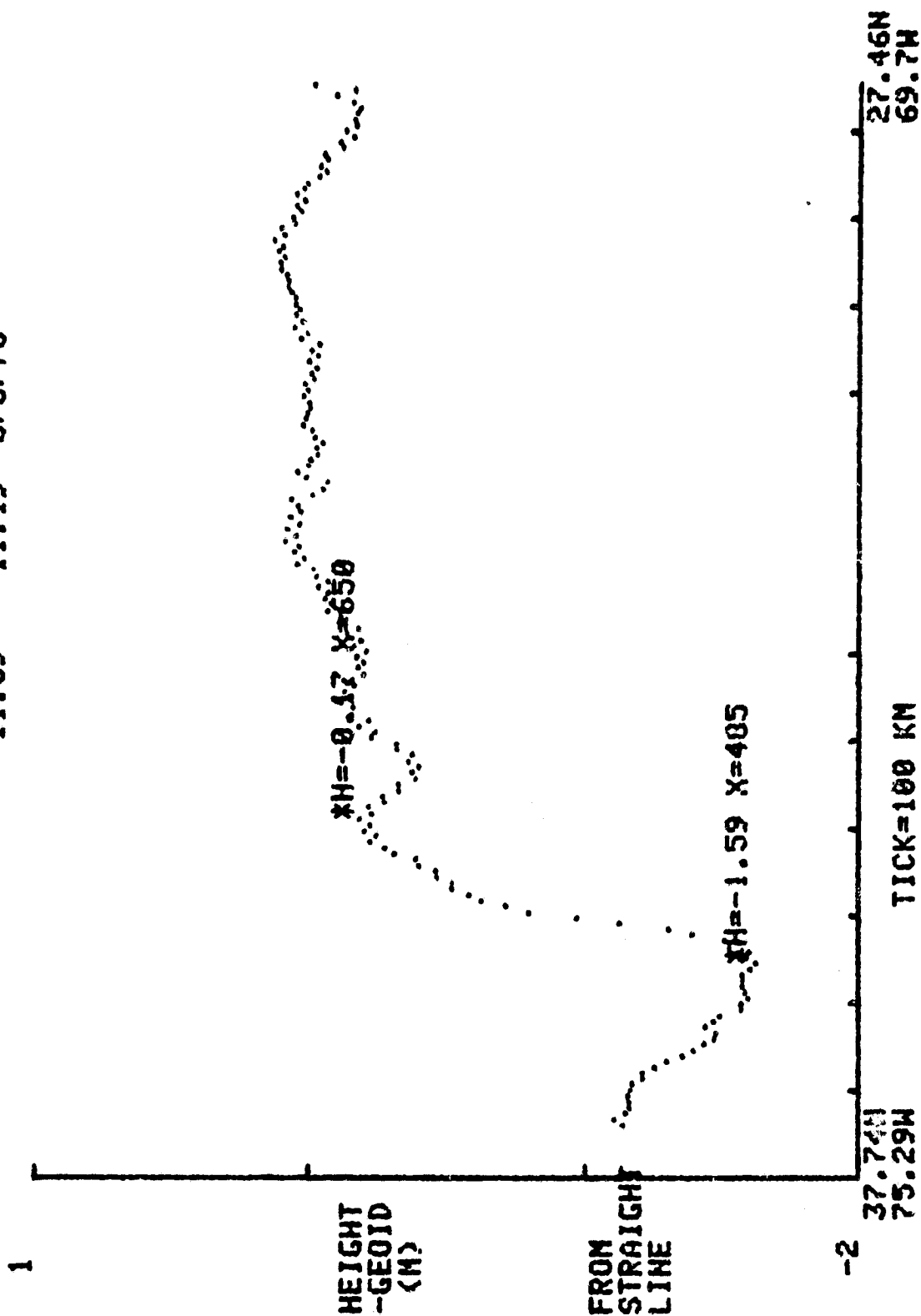


FIGURE 2d

ORBIT 449  
9.69 - 9.83 27/7/78

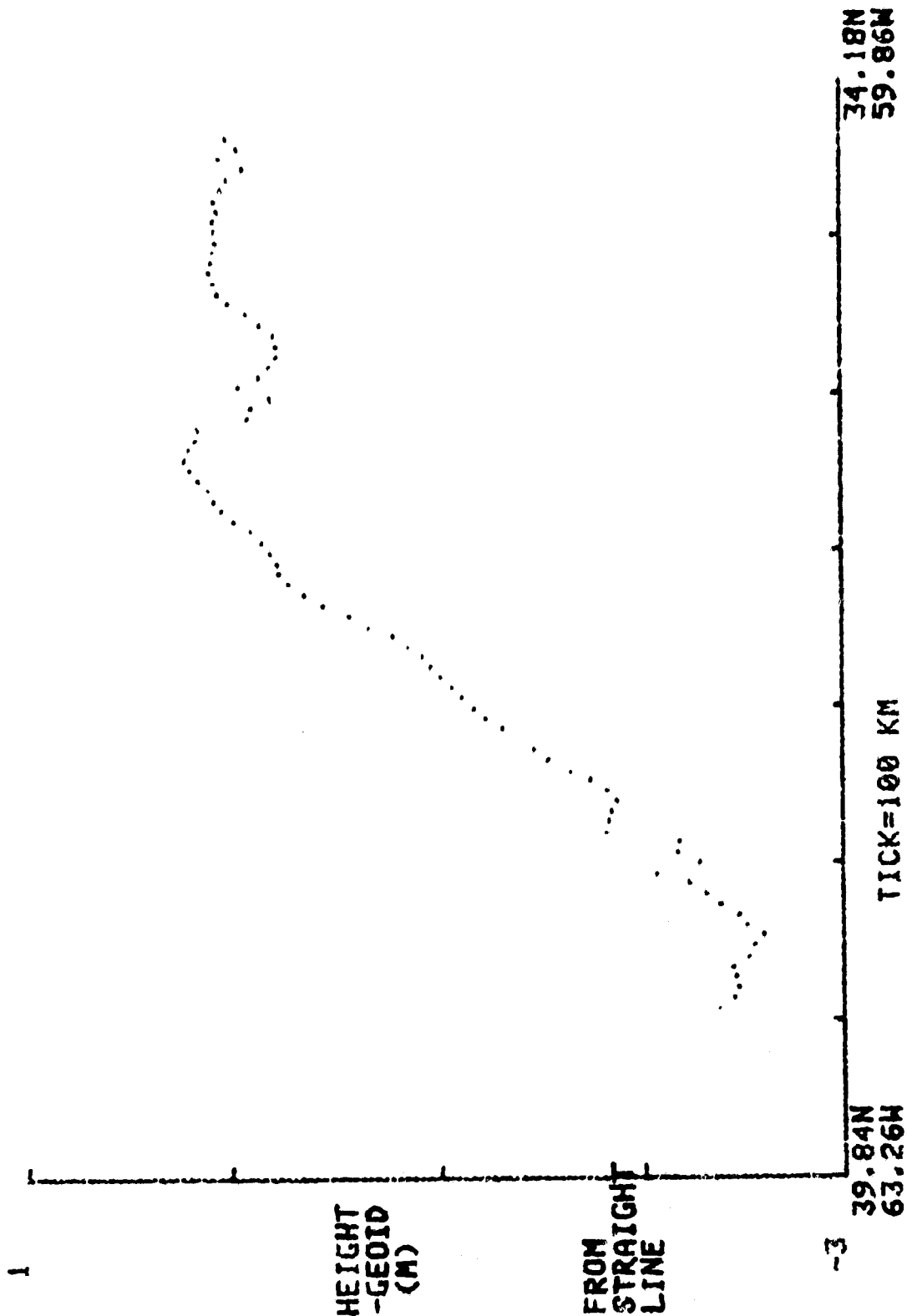


FIGURE 3a

ORBIT 464  
10.85 - 10.96 28/7/78

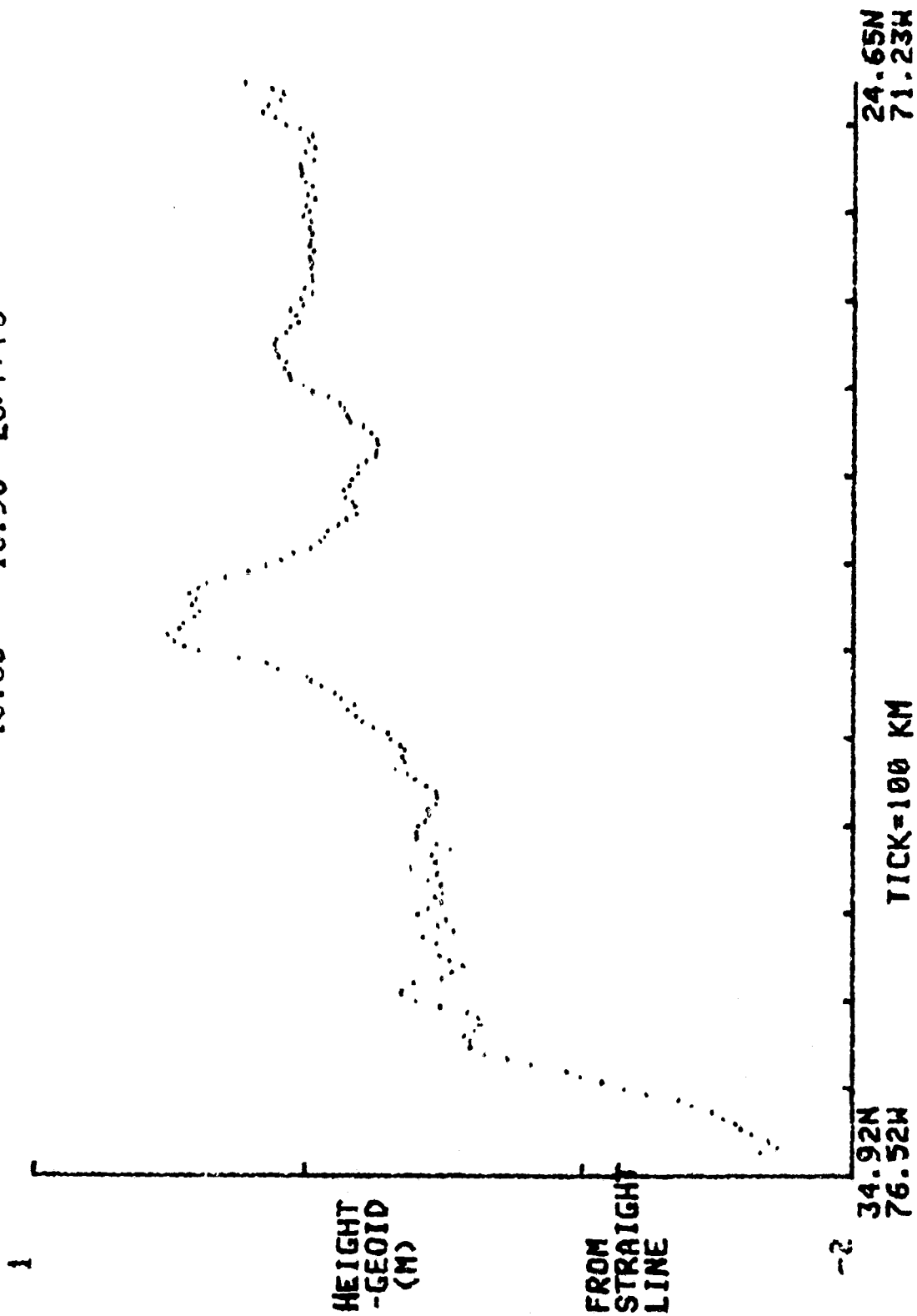


FIGURE 3b

ORBIT 478  
10.33 - 10.45 29/7/78

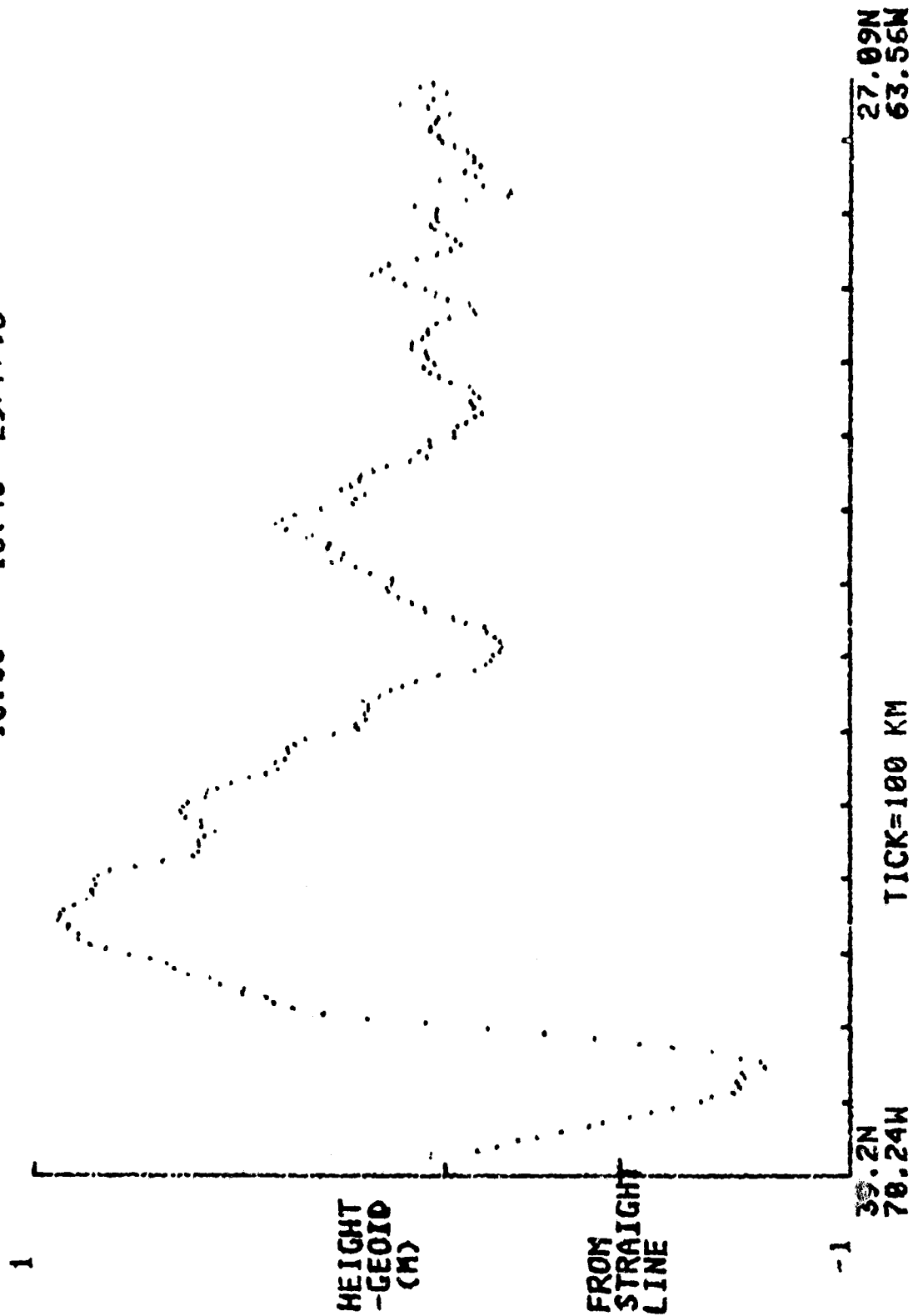


FIGURE 3c

ORBIT 493  
11.46 - 11.57 30/7/78

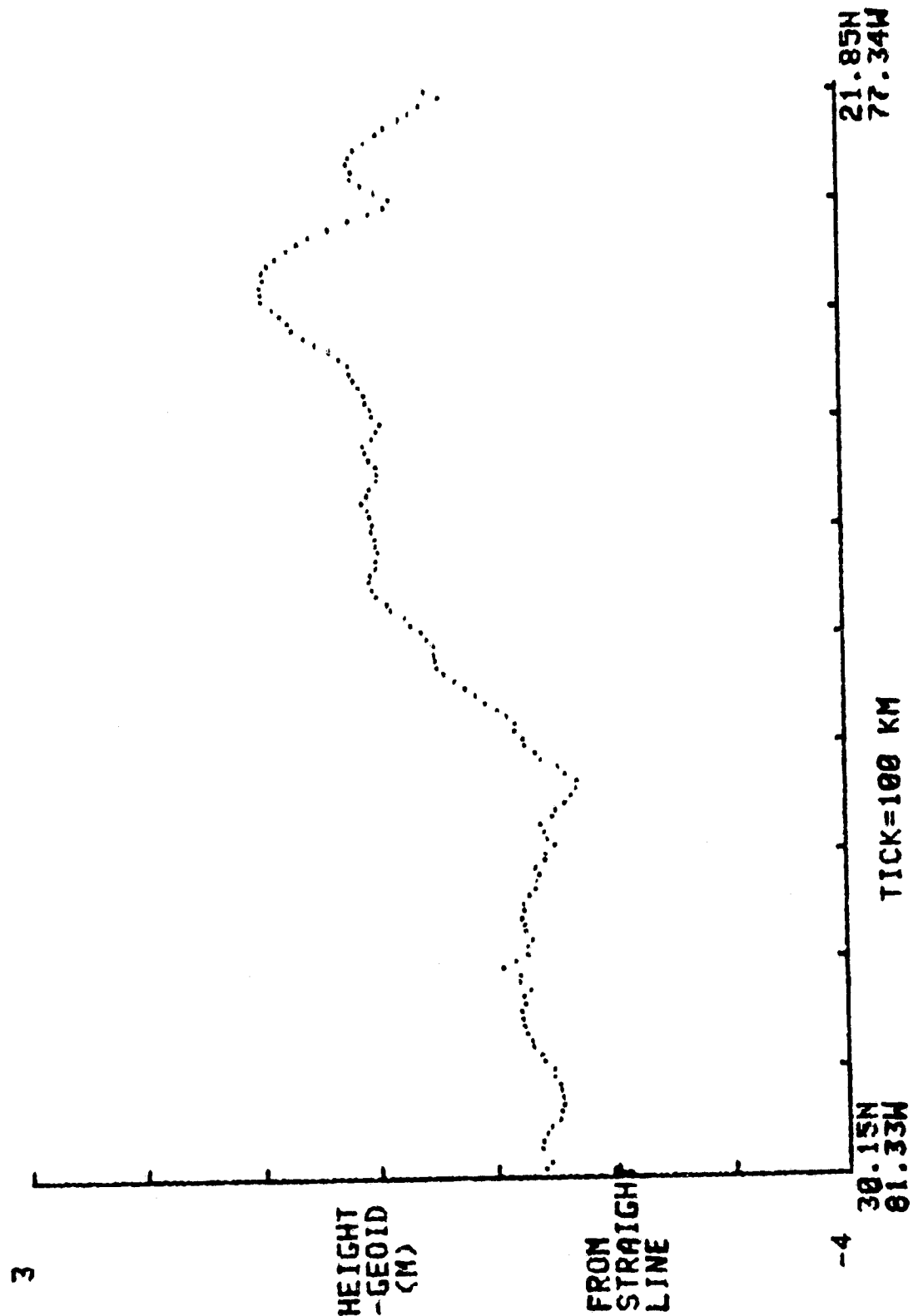


FIGURE 3d

ORBIT 550  
11.09 - 11.19 3/8/78

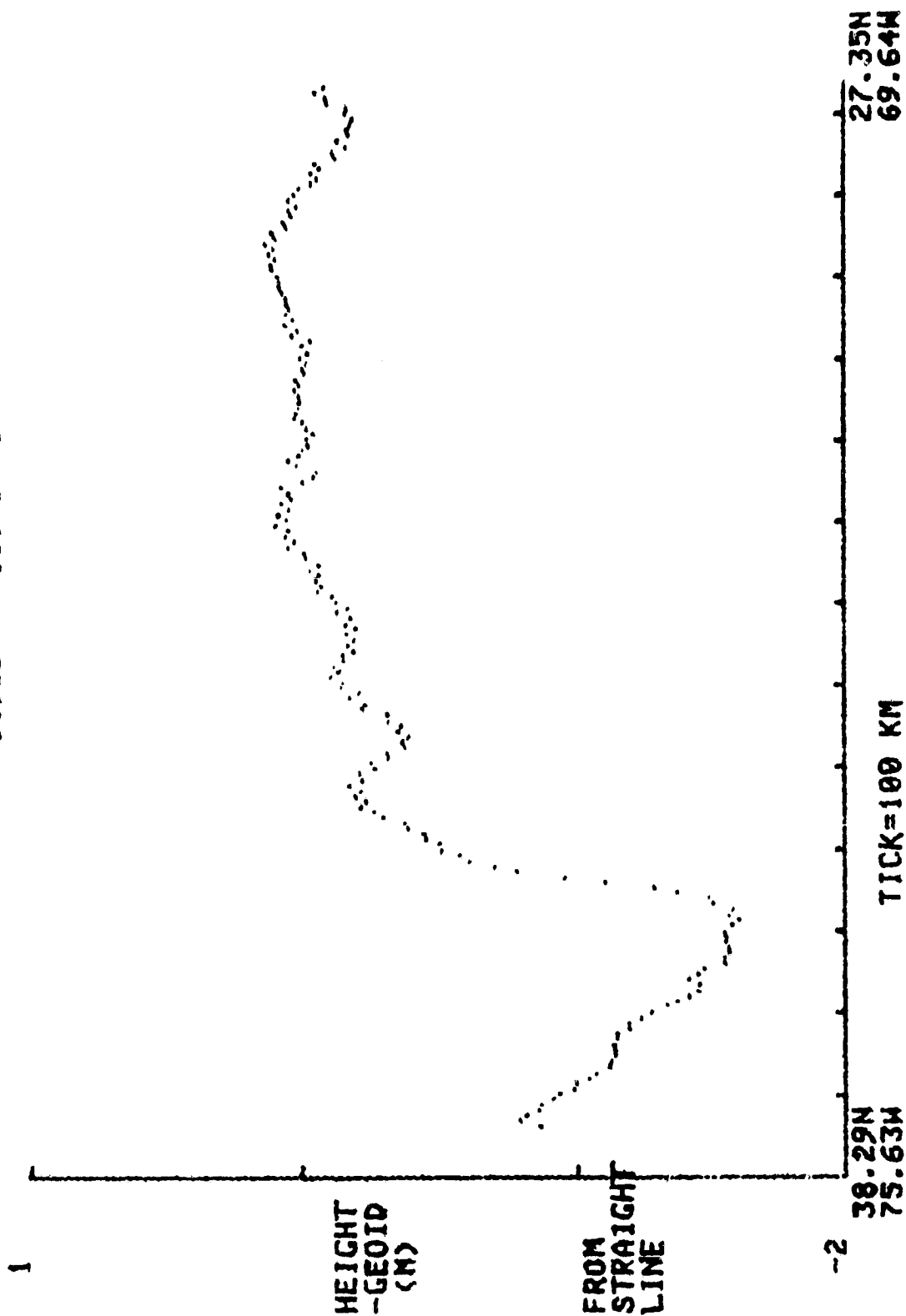


FIGURE 3e



ORBIT 579  
11.72 - 11.8 5/8/78

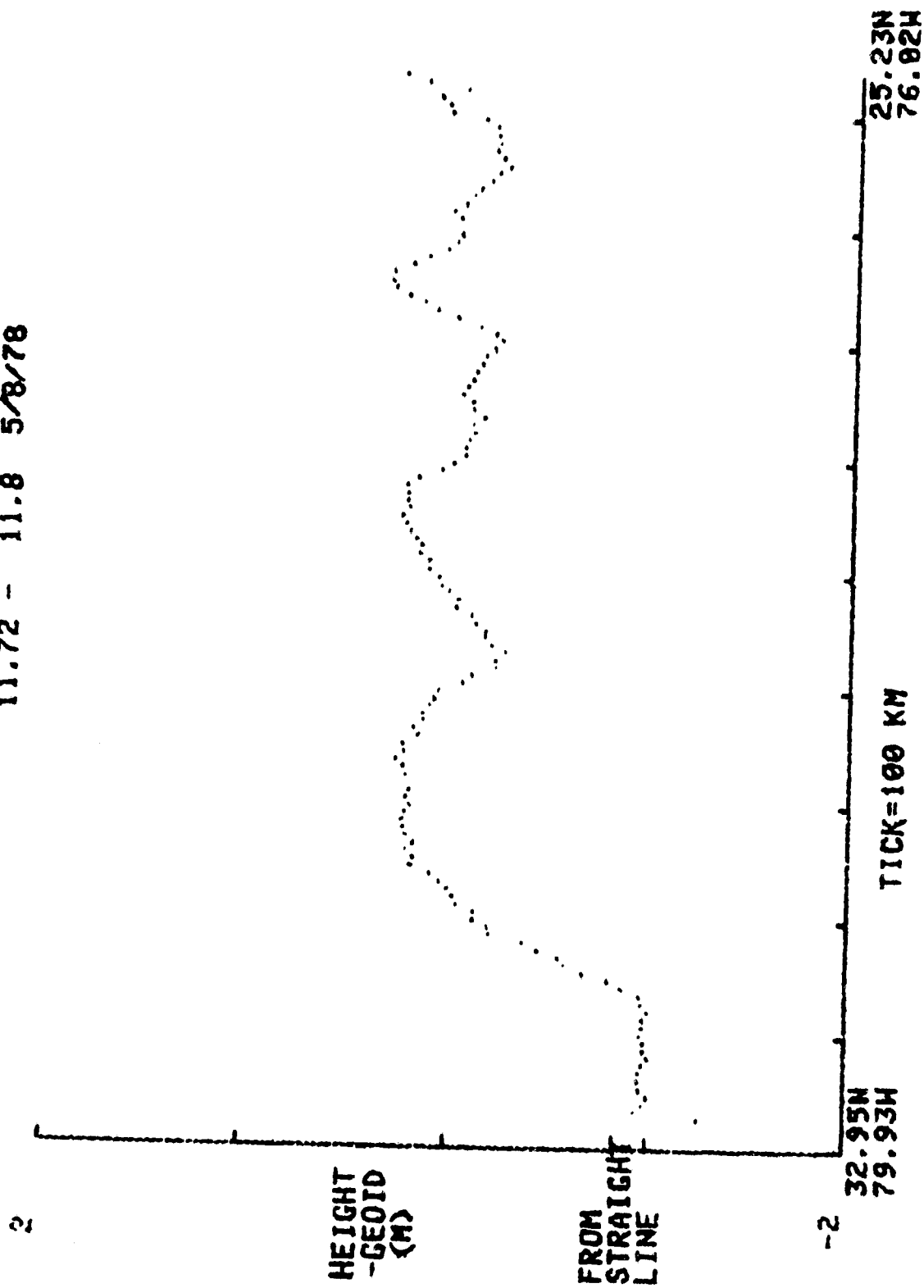


FIGURE 3f

ORBIT 622  
11.79 - 11.98 8/8/78

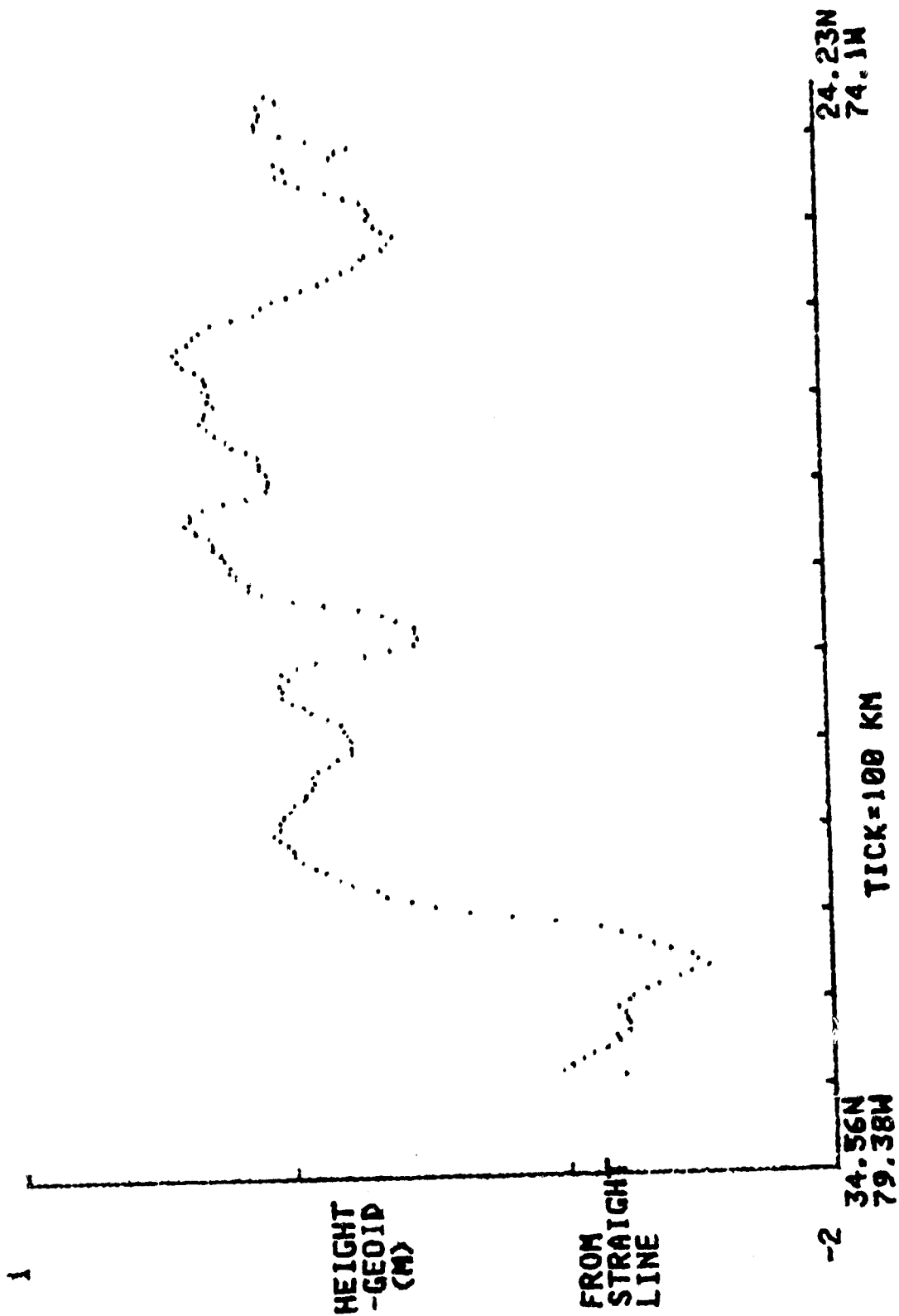


FIGURE 3g

ORBIT 636  
11.32 - 11.44 9/8/78

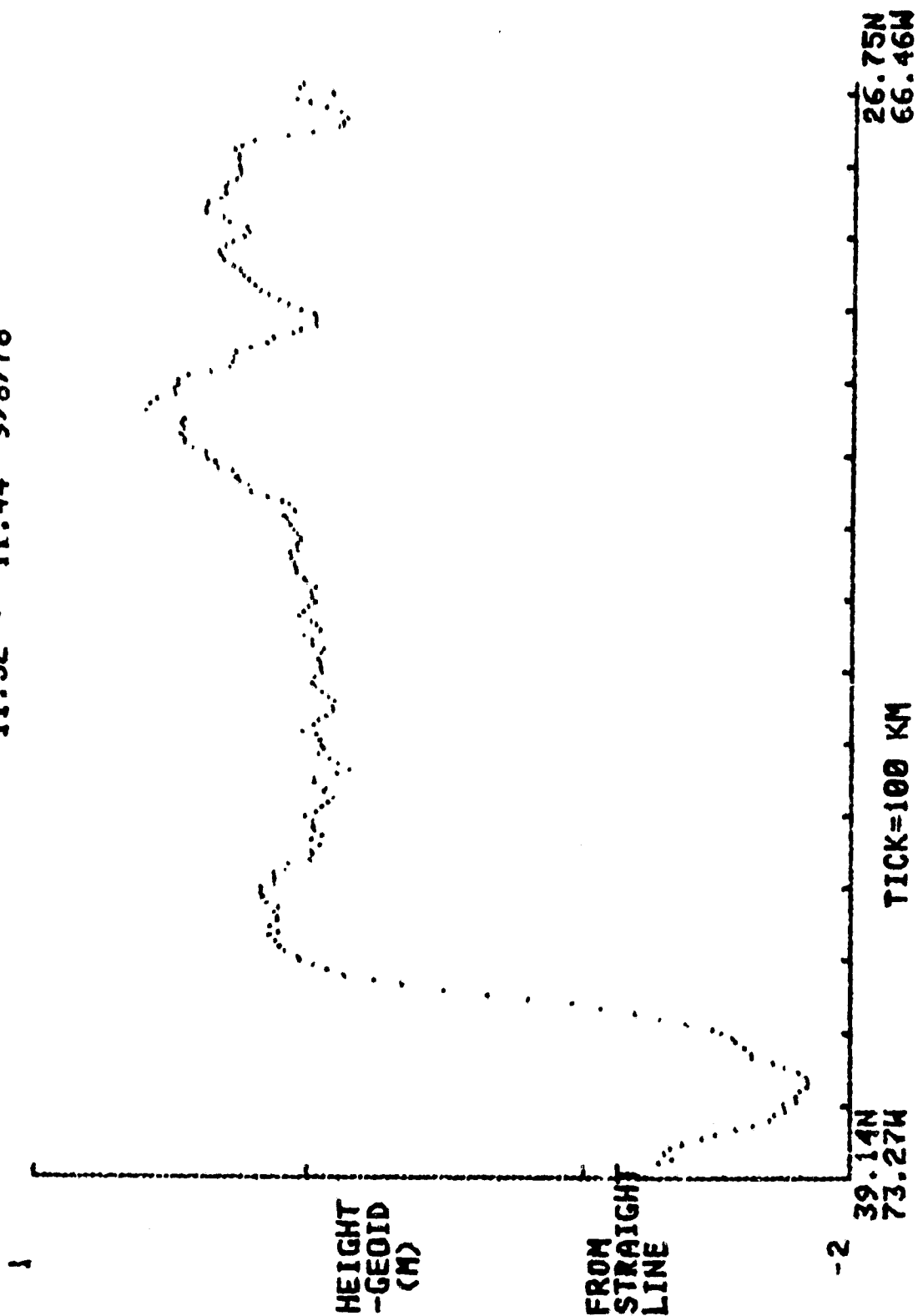


FIGURE 3h

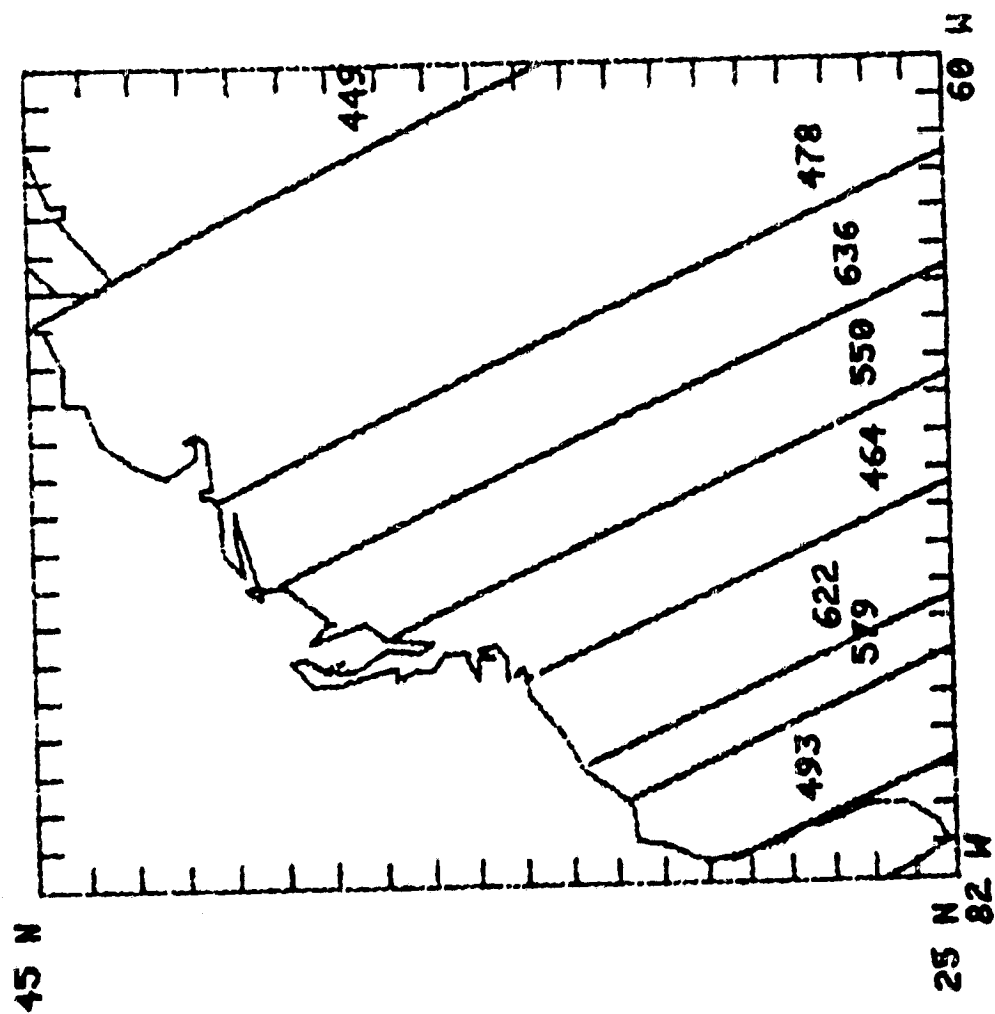


FIGURE 4

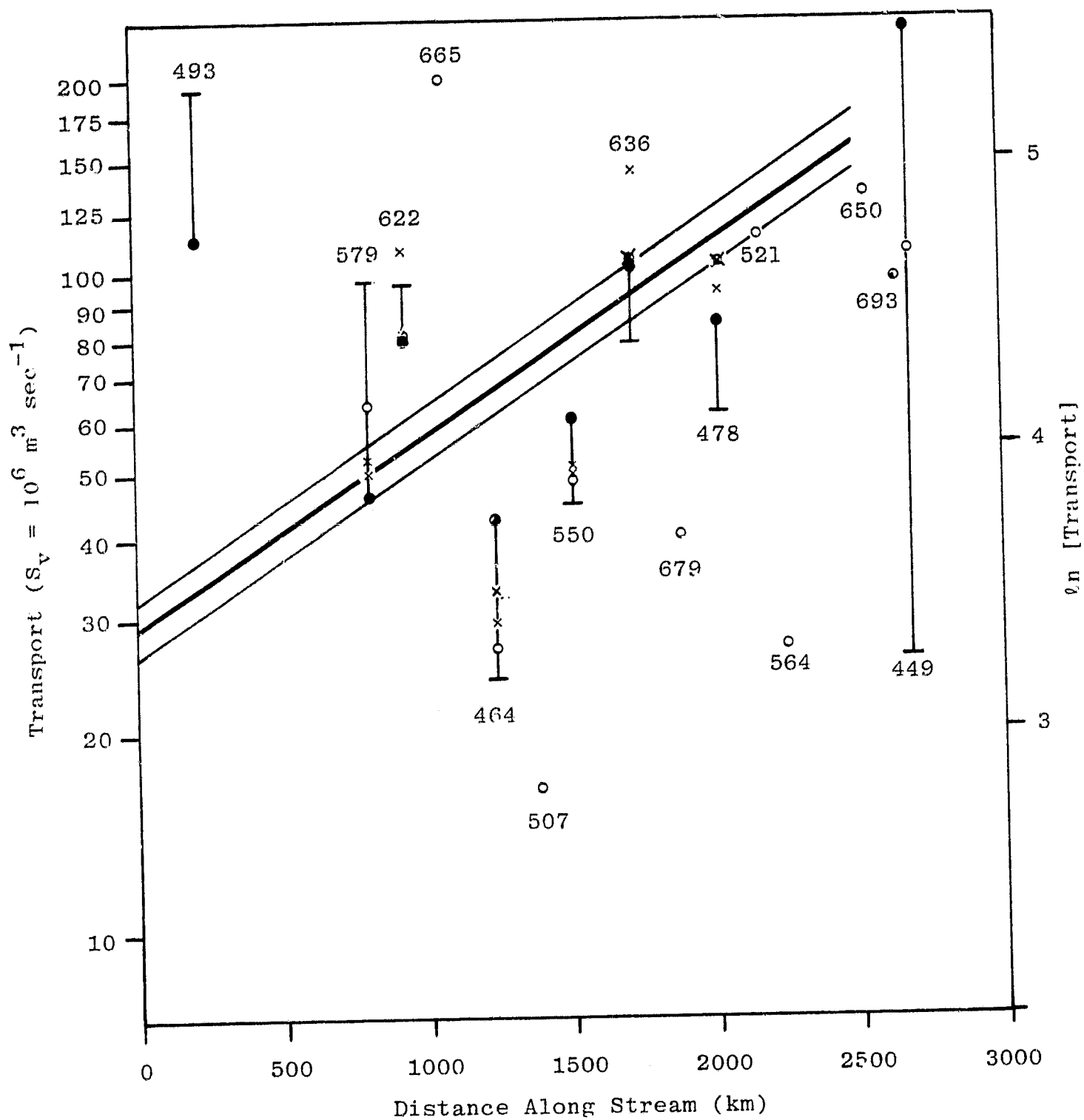


Figure 5a) Ascending Orbits

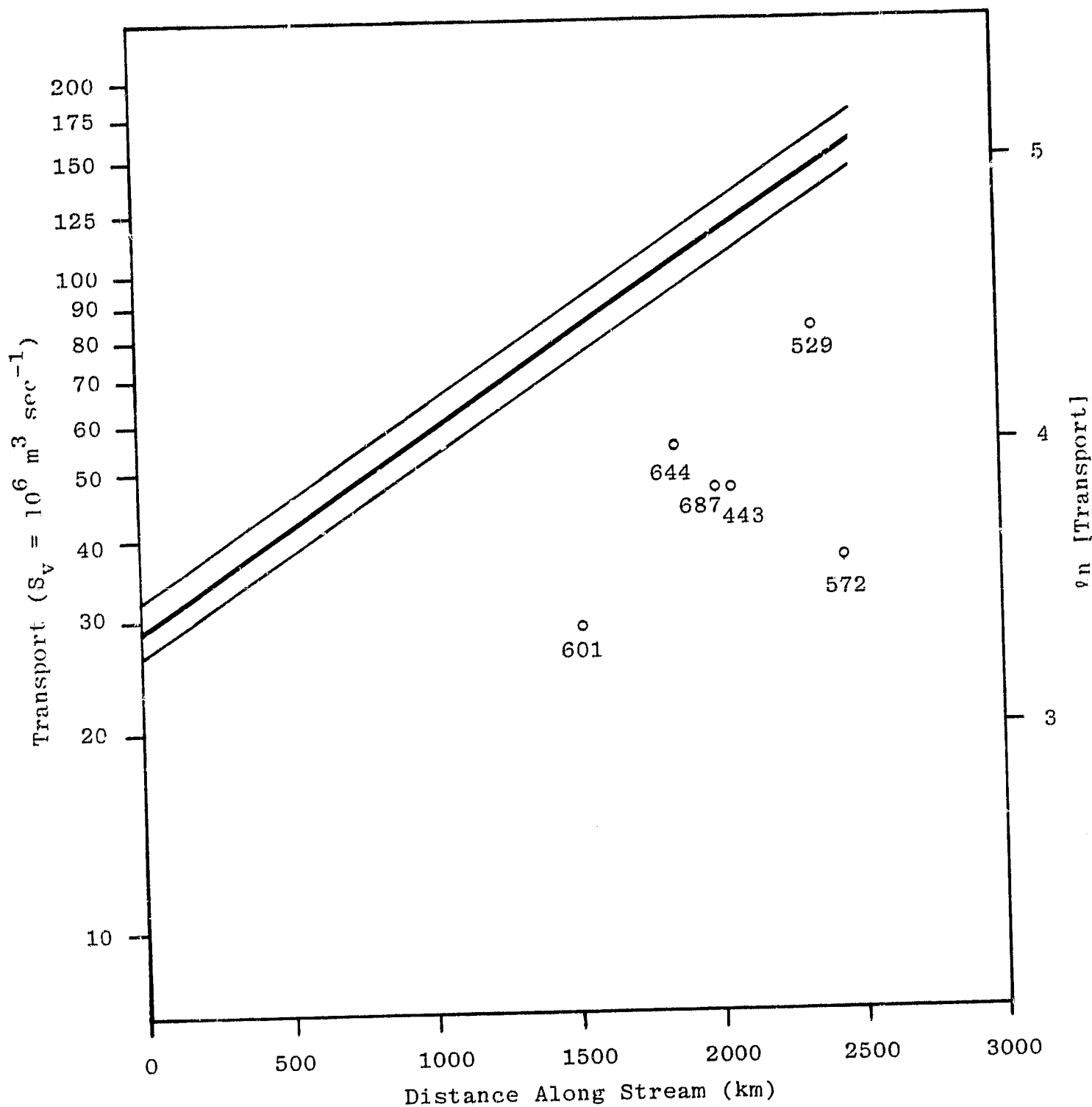


Figure 5b) Descending Orbits

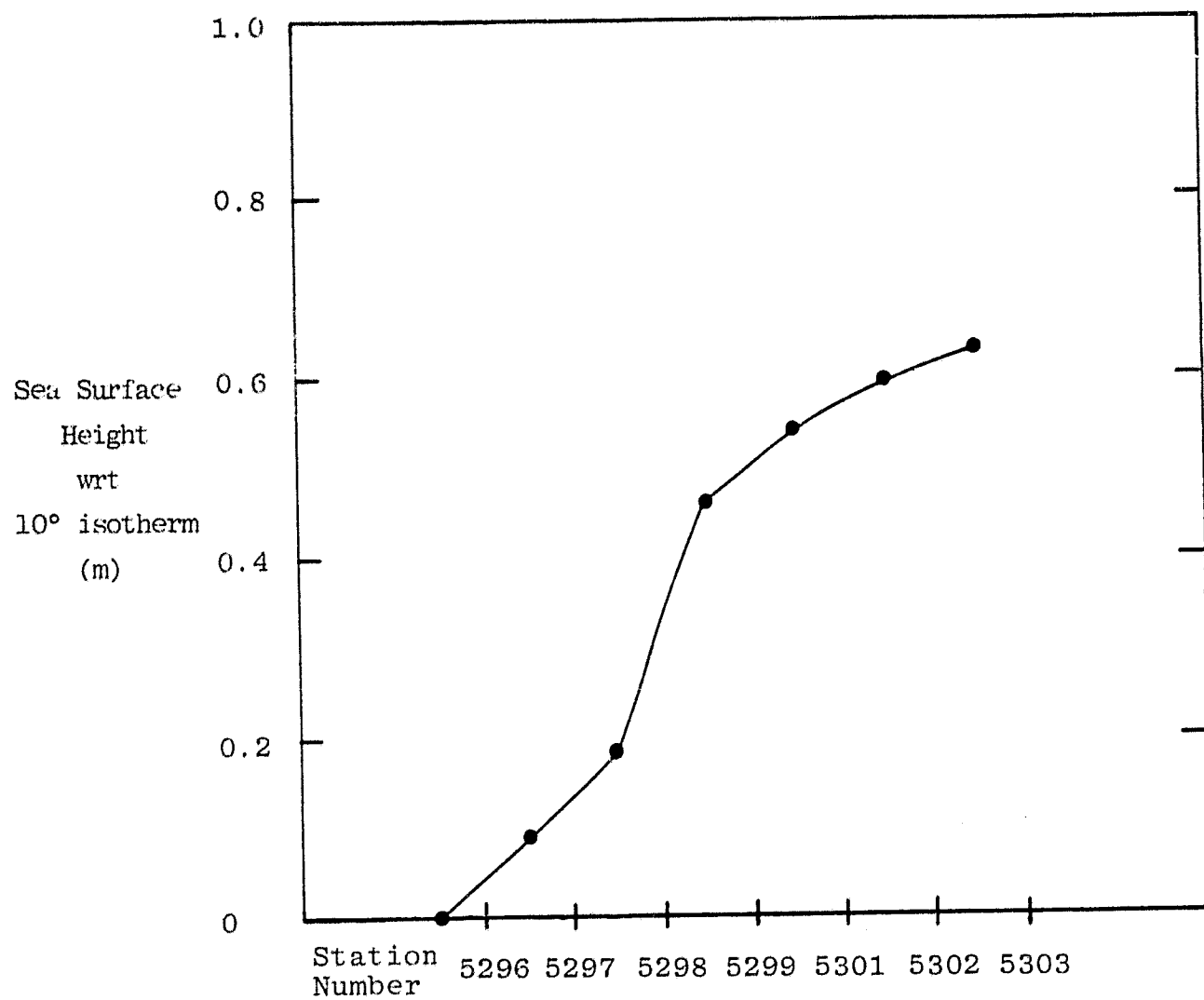


FIGURE 6

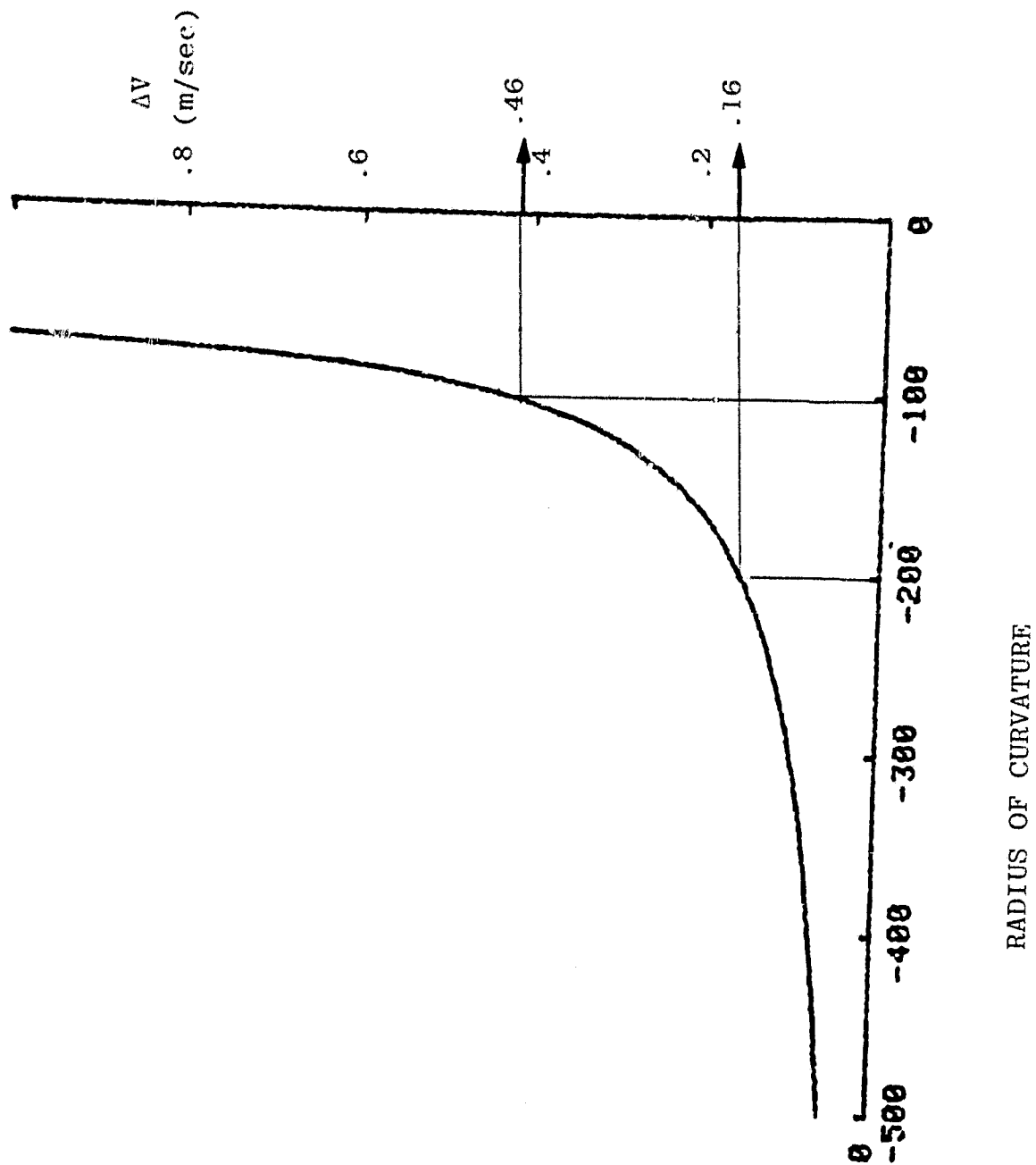


FIGURE 7a



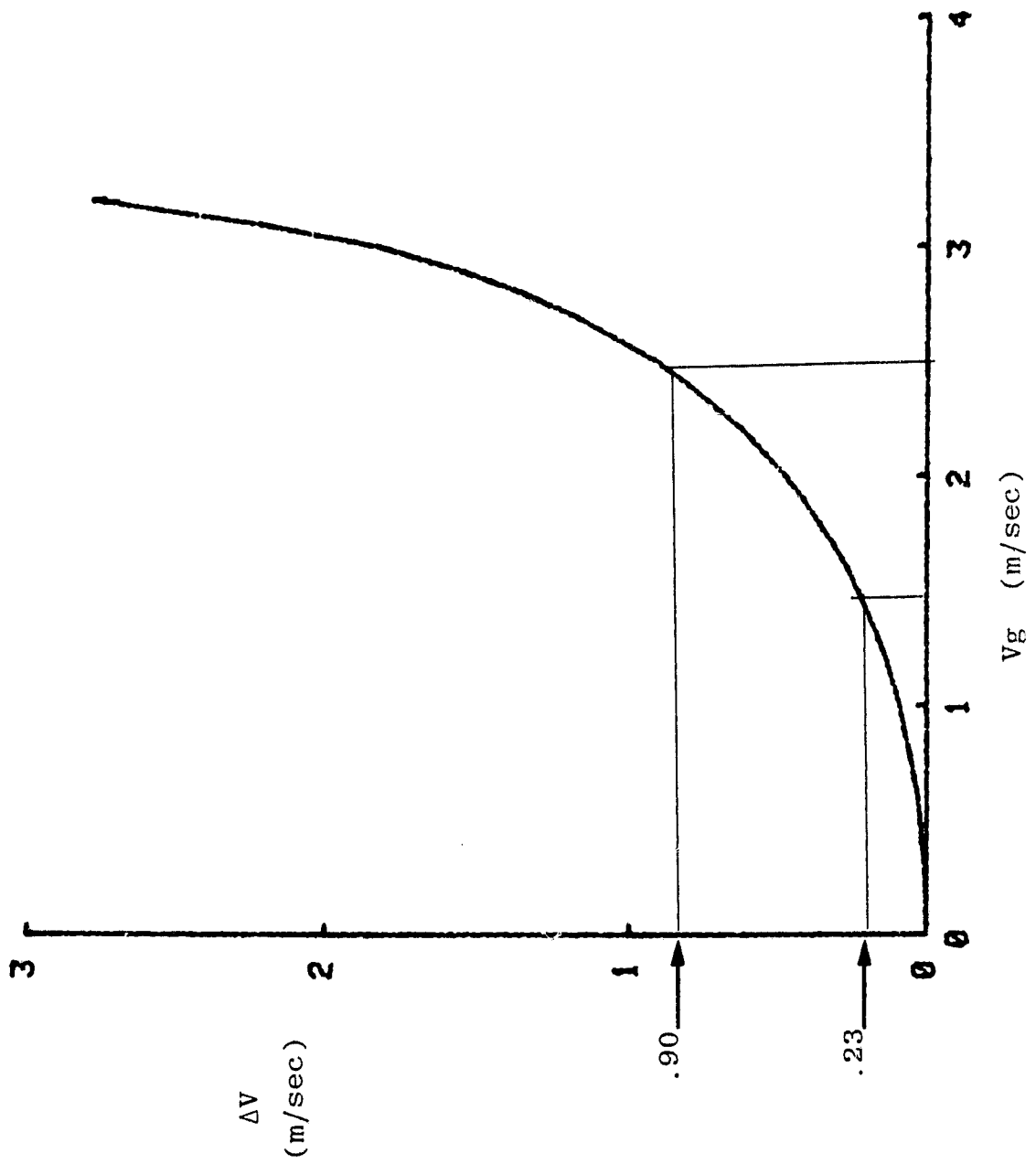


FIGURE 7b